BACK

USING A COMPUTER ALGEBRA SYSTEM TO TEACH QUADRATIC FUNCTIONS

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The following reports on a pilot study for work that will be carried out in the future on children's understanding of quadratic functions using a Computer Algebra System (CAS). The CAS we choose to use is Derive.

Before I begin, I would like to outline my ideological stance. First, I come to CASs with two hats on. As a mathematician I am enthusiastic about Derive. It is fun and exciting and this is very important in mathematics. As an educationalist I must ask sober questions: do CASs improve learning ? if so, then how ? if so, then what evidence do we have for this ? Second, mathematics education (the didactics of mathematics) is not just curriculum analysis, it is also cognitive analysis. It is a sad but true fact that much cognitive curriculum analysis only takes place in the context of primary education. Third, I believe that CASs can do what calculators did 10-15 years ago: cause a resequencing of topics, cause us to question the need to drill some basic skills and improve students' problem solving performance.

At the University of Leeds a group of five teachers and lecturers will use a standard control/experimental group model to examine 14-15 year old mainstream comprehensive school students' understanding of concepts and skills associated with quadratic functions. The students will be drawn from higher and middle set groups. There are three main reasons for choosing this topic and age and ability range of students:-

1) Quadratic functions is a generic topic that links algebraic and graphical concepts. Success in many aspects of higher mathematics depends on students being able to integrate algebraic and graphical representations.

2) Quadratics functions is taught prior to the national examination at 16 years of age. England has a lower post-16 staying on rate than most of its European partners. If CASs can assist students' learning of algebraic and graphical concepts, then they should be introduced as soon as possible.

3) The national examination in mathematics at 16 years of age has three levels (lower, intermediate and higher). Students sit only one of these levels. At present quadratic functions are taught only at the higher level. We wish to examine whether CASs can open up this area of mathematics to pupils in the intermediate level. If there are possibilities here, then this raises the whole question of access to post-16 mathematics.

The pilot study, conducted in the period February - April 1992, used a higher and a middle set group at two state schools both with the full ability range of students. Early work on using Derive for introducing quadratic functions with higher ability students has already been reported (Marshall, 1992). The main purpose of the pilot was to try out the pre-tests, post-tests, instructional materials and an interview schedule so as to make suitable amendments for the larger study to come. The data from the tests and interviews are thus of use only to the research team at the moment.

The instructional material consisted of an A4 booklet with nine pages. I attach, as a sample, the last two pages of this booklet (labelled sheet 4(a) and sheet 4(b)). The overall structure of the instructional material was: setting up Derive (algebra and graphics windows); quadratic graphs (entering expressions and plotting graphs); looking for lowest points; solving equations by graphs; expanding and factorising algebraic expressions; and solving equations algebraically. The larger study to come will include linear equations and graphs as well as quadratic equations and graphs.

The sample pages attached reveal our educational standpoint: to encourage students to use pen and paper as well as the computer; to enable students to work independent of the teacher (this does not undermine the need for a teacher, it merely changes his/her role); and to include a basic section to be completed by all students and an extension section to be attempted by the more able student. The sample pages also reveal shortcomings in the material, for example, the question at the end of item 7 "How would you have solved it without the computer?", produced many humorous remarks but served no particularly useful purpose. These sample pages also reveal an educational weakness with Derive - it does not properly distinguish between an expression and an equation.

Apart from data generated from tests and interviews we are concerned, as a group, as to the strategies students use when doing mathematics with a CAS. In the pilot we videoed the lessons to assist us in examining how students worked. The following are some preliminary observations from the videoed work of the middle set class. I make these points in no particular order:

a) The language of the students is unrefined. It consists, to a large part, of "this", "that" and "there".

b) All of the middle ability group appears able to do the work set.

c) The students do not simply watch the screen, they use their books as directed and to check work.

d) The students use the computer to check predictions they have made.

e) The CAS generates considerable group discussion.

f) The CAS gives rise to the opportunity for students to explain to their fellow students what is happening.

g) Students often work 'backwards' compared with traditional approaches, e.g. explaining how an expression was factorized by the computer. We recorded one interesting hybrid form of explanation where the student uses both sides of the equation to explain the right hand side. Pointing to $x^2 + 7x + 12 = (x + 3)(x + 4)$ the student says, "You divide that one by that one to get that one ($12/4 = 3$) and you take away that one from that one to get that one ($7 - 3 = 4$)."

h) Teacher summaries came from group responses and came after results were discovered (though this results in part from the design of the material).

i) Students can be distracted by the computer and so not perform tasks they could otherwise do, e.g. one students, who was trying to answer his own query on where $f(x) = (x - 9)(x + 7)$ crosses the y-axis, was unable to make the substitution $x = 0$.

j) Students were not really interested in exploring what complex number solutions were. This was not the case with the students in the higher ability set.

Finally, I would like to acknowledge that the work reported on here is not entirely my own, but represents activities undertaken by all of the Leeds group, in particular Mark Hunter and Paul Marshall.

Reference

Marshall, P., (1992), 'Symbol Manipulators at GCSE', *Mathematics in School* **21**(1), 14-15.

Sample Instruction sheets

GCSE Computer Algebra Sheet 4(a)

Using the soLve option

The computer is capable of solving equations using the soLve option. You need to define the left hand side of the equation (it assumes the right hand side is zero), then press <L> for solve.

1 Use <A>uthor to define $x^2 + 3x - 4$.

2) Press <L> Write down the two solutions.

3) Check, by plotting, that the two solutions are correct.

4) <F>actorize the same expression. Write down what you notice. Can you explain this ?

5) For each of the following, <F>actorize the expression and soLve the equation.

Put your answers in a table.

Equation	Factors	Solutions
a) $x^2 + x + 30 = 0$	)( )	x = or
b) $x^2 + 3x - 54 = 0$		
c) $x^2 - 7x - 18 = 0$		
d) $x^2 - 7x + 12 = 0$		
e) $x^2 - 11x + 30 = 0$		

6) Solve the following equations without using the computer:

a) $x^2 + 3x + 2 = 0$

b) $x^2 + 8x + 15 = 0$

c) $x^2 - 9x + 20 = 0$

d) $x^2 + 2x - 35 = 0$

e) $x^2 - 7x + 18 = 0$

7) Try solving this equation using the computer

$$x^2 + 2x - 6 = 0$$

You should get a rather strange looking answer with square roots in it. This is the exact answer. You can get an approximate answer if you change modes. Press <O>ptions and <P>recision. Use the space bar to change to mixed mode. *Highlight* the equation using the arrow keys. SoLve it again. Note down your solutions.

How would you have solved it without the computer ?

8) Extensions

a) How can you tell by looking at a quadratic expression whether or not it will factorize.

b) How can you tell by looking at a quadratic equation whether or not it has 2, 1 or 0 solutions ?

c) Investigate equations that have $2x^2$ or $3x^2$ instead of x^2.

d) Can cubic equations be factorized / solved ?