BACK

Derive and 16-19 Mathematics : A Blessing and not a Curse.

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Introduction

In order to illustrate the main thesis of this paper I wish to present a set of Derive teaching materials which can be used directly by students. The set consists of 10 MTH files on disk together with the hard-copies of the contents of these files which are included as appendices to this paper and which are intended to be used in conjunction with the MTH files.

It is assumed that students using these materials have already met the basic facilities of Derive in an introductory course.

The target group for these materials are mathematics students aged 16-19 taking an A level Further Mathematics course (Pre-University).

These materials have been assembled in order to show the specific contribution that Derive can make through the application of its unique facilities. The facilities specifically drawn upon which are unique to computer algebra systems and to Derive in particular are as follows.

1. Symbolic manipulation;

2. A matrix handling capability ;

3. A Lisp like functional programming language;

4. A flexible Calculus/Algebra/Graphics interface;

5. A large Integer handling capacity together with exact arithmetic;

6. A facility for declaring, manipulating and combining functions using normal mathematical notation;

7. A Windows screen environment;

8. A capacity to combine text and computable expressions within MTH files.

Some simple summation and divisibility problems are explored in SUMS.MTH and DIVISIBLE.MTH. (An interesting parallel emerges between the process of proof by induction of divisibilty claims and the Euclidean Algorithm while working on these problems with Derive.) MOBIUS.MTH shows how Derive can really come into own in this respect through the investigation of a parameterised Mobius map.

2. The mathematical modelling power offered to students by Derive is illustrated by the files DONSDATE.MTH, SEALPOP.MTH and STOCK.MTH. The first two files both involve matrix transition models. The difficulty of extracting information from such models has always militated against their exploration at an elementary level. Through the simplicity of its functional language Derive enables the rapid manipulation of such models and the extraction of graphical information from them.

DONSDATE.MTH which explores a model of the potential dating behaviour of Don Giovanni from Mozart's opera is included partly to make a humourous connection with the International Spring School's Venue. However it has a serious purpose too, in that it shows how information can be extracted through matrix iteration using Derive which normally would have required an eigenvalue/eigenvector analysis (which is shown to be possible with Derive too).

SEALPOP.MTH more seriously shows how a cohort population model invented by the British zoologist Leslie which is potentially intractable to analysis can be explored both numerically and graphically using Derive .

The applicability of matrix models is wide. Derive now enables this useful modelling method to be meaningfully exploited at the 16-19 level.

STOCK.MTH presents a more familiar functional model but with 2 degrees of freedom rather than the usual 1 degree of freedom. The objective in this exercise is to illustrate the techniques of graphical analysis through the use of 3D-Plot ,the VECTOR function and 2D-Plot which enable the optimal values of the model to be determined without resort to calculus. The results are compared with a Derive assisted calculus-based approach too.

The net effect of Derive in this respect is to enable a new set of modelling tools to be employed at a more elementary level of the education system. The ease of manipulation of these sometimes complex models with Derive allows us to redirect our attention in modelling to (a) the process of formulation and the underlying assumptions made and to (b) the nature of the information that we can extract from the model and its relationship to these assumptions.

3. Many of the graphical, algebraic and calculus-based techniques presented by Derive are employed to good effect in the exploration of the logistic function on the route to chaos in LOGISTIC.MTH. An exploratory approach is employed throughout with considerable scope left for students to formulate their own ideas and test them out.

1. In formal reasoning it allows us to focus upon the global pattern of the proof as against the myriad detail of its implementation.

2. In mathematical modelling it enables us to extract information from the models we set up in a rapid and graphic way and thus it redirects our attention to the process of formulation of the model and the interpretation and validation of the information that it produces. In addition it expands the range of modelling tools available to the 16-19 year old.

3. It enables students to explore empirically new and old mathematical theories and experimentally to test conjectures in an immediate and direct way.

4. Derive , by its very functioning, raises a host of new questions for discussion and analysis which are new to the Sixth Form mathematics classroom.

5. Finally, and not least from a pedagogical point of view, Derive offers a new pedagogic form in the MTH file. The possibility of combining explanatory text and functioning expressions in a consistent way in a single artifact represents a new possibility for mathematics tuition.

Examples of such artifacts I have attempted to produce in the following Derive mathematics tutorials, where the hard-copies and the MTH files on disk are intended to be used together assisted by the JUMP command.

Acknowledgements

Douglas Quadling : Statistics and Probability ,CUP
(for Don Giovanni)
Andrew Paterson : Differential Equations and Numerical Analysis, CUP.
(for Burmese beetles)
Predrag Cvitanovic : Universality in Chaos, Adam Hilger
(for Logistic function)
Kenneth Rosen : Elementary Number Theory and its Applications, Addison Wesley
(for pseudoprimes)
Keith Devlin : Mathematics:The New Golden Age, Penguin
(for Mersenne and Fermat)
Franco Vivaldi : QMWC University of London
(for the parameterised Mobius map)
Brian Conolly : QMWC University of London
(for the stockout inventory model)

SUMS.MTH

1: "AN EXAMPLE OF A DERIVE ASSISTED PROOF BY INDUCTION"

2: "Investigate the sum of the first n odd integers. (Use Calculus Sum)"

3: $\sum_{i=1}^{n} (2\,i - 1)$

4: $\text{VECTOR}\left[\sum_{i=1}^{n} (2\quad - 1),\ n,\ 1,\ 10\right]$

5: [1, 4, 9, 16, 25, 36, 49, 64, 81, 100]

6: "This suggests the following conjecture."

7: $\sum_{i=1}^{n} (2\,i - 1) = n^2$

8: "Suppose this is so. The sum of the first n+1 odd integers must thus be:"

9: $n^2 + (2\,(n + 1) - 1)$

10: "This simplifies and factors as follows."

11: $n^2 + 2\,n + 1$

12: $(n + 1)^2$

13: "And Manage Substitute on expression 7 gives"

14: $\sum_{i=1}^{n+1} (2 \quad -1) = (n+1)^2$

15: "By induction on n this must thus be true for all positive integers."

16: "INVESTIGATE"

17: $\sum_{i=1}^{n} \frac{1}{i\,(i+1)}$

18: "WHEN YOU'VE FINISHED TRANSFER LOAD DIVISIBLE.MTH"

DIVISIBLE.MTH

1: "A FURTHER EXAMPLE OF A DERIVE ASSISTED PROOF BY INDUCTION"

"Is the sum of the cubes of 3 consecutive positive integers "

3: "always divisible by 9 ?"

4: $n^3 + (n + 1)^3 + (n + 2)^3$

5: $\text{VECTOR}\left[\dfrac{n^3 + (n + 1)^3 + (n + 2)^3}{9}\right]$

6: [4, 11, 24, 45, 76, 119, 176, 249, 340, 451]

7: "This suggests the result. Suppose its true up to n."

8: "Manage substitute on expression 4 gives:

9: $(n + 1)^3 + (n + 1 + 1)^3 + (n + 1 + 2)^3$

10: "Using Build we can construct the difference between #9and #4."

11: $(n + 1)^3 + (n + 1 + 1)^3 + (n + 1 + 2)^3 - (n^3 + (n + 1)^3 + (n + 2)^3$

12: "This simplifies to :"

13: $9\,(n^2 + 3\,n + 3)$

14: "Given "

15: $n^3 + (n + 1)^3 + (n + 2)^3$

16: "is divisible by 9 ,"

17: $(n + 1)^3 + (n + 1 + 1)^3 + (n + 1 + 2)^3$

18: "must thus also be divisible by 9 and so by induction on n, the"

19: "statement must be true for all positive integers."

20: "INVESTIGATE"

21: $(8^n + 6)$ "is divisble by 14."

22: $(9^n + 7)$ "is divisible by 8"

23: $(n^3 - n)$ "is divisible by 3"

24: $(n^5 - n^3$ "is divisible by 24"

25: $(5^n - 4 n - 1)$ "is divisible by 4"

26: $(2^{n + 3} + 3^{n - 1}$ "is divisible by 5"

27: "For #24,#25 and #26 a little extra thought is needed."

28: "WHEN YOU'VE FINISHED TRANSFER LOAD MOBIUS.MTH FOR AN EXTRA CHALLENGE"

MOBIUS.MTH

1: "The following Mobius Map has an interesting property."

2: $$F(z) := \frac{(\mu + 1)\, z + (\mu - 1)}{(\mu - 1)\, z + (\mu + 1}$$

3: "Use Derive to explore the following function compositions."

4: F (F (z))

5: F (F (F (z)))

6: F (F (F (F (z))))

7: F (F (F (F (F (z)))))

8: "By Simplifying, highlighting the numerators and denominators"

9: "and Expanding and Factoring, the resulting expressions can be "

10: "manipulated into a neat form."

11: "Suggest an expression for the nth composition of F with itself"

12: "and use Derive to help you prove this by induction."

DONSDATE.MTH

1: "TRANSFER LOAD UTILITY VECTOR.MTH BEFORE LOADING THIS FILE"

2: "d represents the Markov transition matrix for the"

3: "dating behaviour of Don Giovanni from Mozart's opera."

4: $$d := \begin{bmatrix} 0 & \frac{3}{4} & \frac{5}{6} \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{6} \end{bmatrix}$$

5: "p represents the initial outcome probability matrix for"

6: "the Don going out with Anna, Elvira or Zerlina."

7: $$p := \begin{matrix} 1 \\ 0 \\ 0 \end{matrix}$$

8: "So initially the Don goes out with Anna. "

9: "By tossing a coin, pulling a card from a pack of playing cards"

10: "and noting its suit or rolling a die the Don decides who he "

11: "will date next. This procedure is represented by the transition"

12: "matrix. By iterating the linear map defined by d upon p the"

```
13:  "outcome probability matrix can be determined for successive"

14:  "evenings."

15:  DONS_DATE_EVENING (n) := ITERATE (d • q, q, p, n)

16:  "Try the following example by highlighting and using approX."

     "Thus determine the respective probabilities with which the"

18:  "Don will date Anna, Elvira or Zerlina 3 evenings after Anna."

19:  DONS_DATE_EVENING (3)

20:  "Who will the Don date most in the long run ?"

21:  "ApproXing the following expression will enable you to determine this."

22:  ITERATES (d • q, q, p, 60)

23:  "A traditional eigenvalue/eigenvector analysis of this Markov model"

24:  "can also be accomplished with Derive to establish similar results."

25:  "(The following assumes knowledge of the algebraic properties"

26:  "of Markov models.)"

27:  "Try the following expressions."

28:  EIGENVALUES (d)

29:  EXACT_EIGENVECTOR (d, 1)

30:  "Use Manage Substitute to replace @1 by an appropriate expression"

     "to find the precise steady outcome probability matrix."
```

SEALPOP.MTH

1: "m is the Leslie Transition Matrix for the grey seal population "

2: "on Farne Island (1978). 'Journal of Applied Ecology 15'"

3: $$m := \begin{bmatrix} 0 & 0 & 0 & 0 & 0.08 & 0.28 & 0.42 \\ 0.66 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.93 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.93 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.93 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.935 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.935 & 0.935 \end{bmatrix}$$

4: "p represents an initial population vector showing the population"

5: "of the Island uniformly distributed between the generations."

6: "NOTE: generation 7 represents the seals over the age of 6 years."

7: $$p := \begin{bmatrix} 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 20 \end{bmatrix}$$

8: "By iterating the linear map defined by m upon p the population"

9: "vectors for successive years can be obtained."

```
10:  SEAL_POP_YEAR (n) := ITERATE (m • q, q, p, n)

11:  "Try the following example by highlighting and using approX."

12:  SEAL_POP_YEAR (100)

13:  "How the size of the different generations vary can also be investigated."

14:  "In the following function q represents the generation number 1,2,.,7 and"

15:  "y represents the number of years over which change is to take place,"

16:  GENERATION (q, y) := VECTOR ([n, ELEMENT (ELEMENT (ITERATES (m • q, q, p, y), n), q, 1)], n,~

     ~ 1, y)

17:  "Try the following example by highlighting and using approX."

18:  GENERATION (1, 20)

19:  "You will note that the GENERATION function generates a matrix of"

20:  "coordinates. These coordinates can be displayed using 2-D Plot."

21:  "However, set up a working screen before attempting this 2D-Plot."

22:  "SETTING UP A WORKING SCREEN"

23:  "Change to graphics display mode. Split Window 1 horizontally."

24:  "Designate Window 2 for 2D-Plot. Configure Window 2 as follows."

25:  "SCALE: X:3,Y:100; MOVE: X:10,Y:100; CENTRE;"

26:  "OPTIONS STATE Mode : Connected."
```

```
27:  "GRAPHICAL ANALYSIS"

28:  "2D-Plot the matrix of coordinates previously obtained."

29:  "The simultaneous plot of such sets of coordinates can give a graphic"

30:  "illustration of the development of the population structure over time."

31:  "This can be efficiently done by approXing the following expression"

32:  "and 2D-Plotting the result."

33:  VECTOR (GENERATION (g, 20), g, 1, 7)

34:  "Finally, you might like to try to investigate the effects of editing the"

35:  "Leslie Matrix. For example, you might note that a culling policy may be"

36:  "appropriate. This would correspond to altering the (2,1) element in m."

37:  "Find a value for this element which will give a stable population."

38:  "EXTENSIONS"

39:  "Burmese beetles (Bernadelli 1941) live at most 3 years. They breed"

40:  "at the end of their third year, each beetle laying eggs which will"

41:  "produce 6 new beetles at the beginning of the new year. Only 1/3 of the"

42:  "new beetles survive to the end of their first year. By the end of"

43:  "the second year 1/2 of these survivors have died.

44:  "Develop and investigate a Leslie model for Bernadelli's Burmese beetles."
```

Screen Layout for SEALPOP.MTH

```
54:  GENERATION (5, 20)

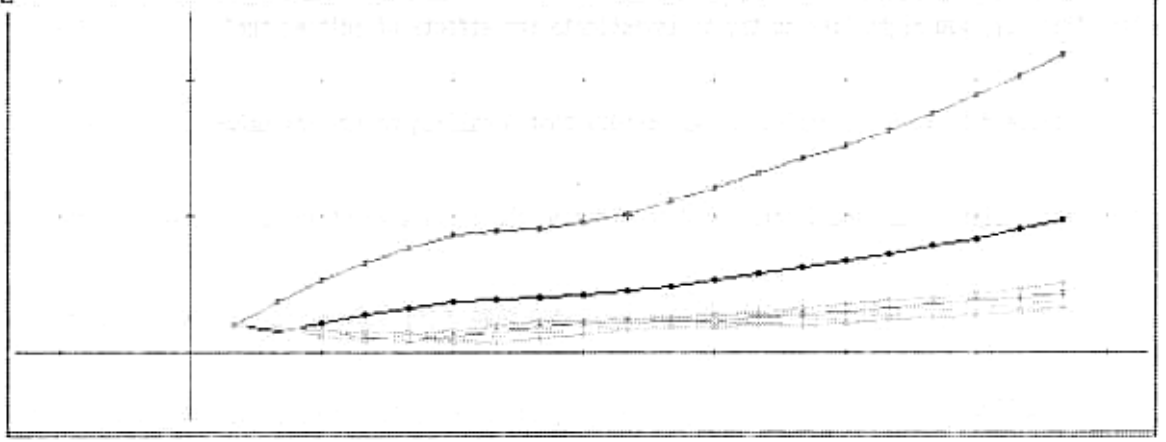
55:  [[1, 20], [2, 18.6], [3, 17.298], [4, 16.0871], [5, 10.6175], [6, 8.28165

56:  GENERATION (6, 20)

57:  [[1, 20], [2, 18.7], [3, 17.391], [4, 16.1736], [5, 15.0414], [6, 9.92736

58:  GENERATION (7, 20)

59:  [[1, 20], [2, 37.4], [3, 52.4535], [4, 65.3046], [5, 76.1821], [6, 85.294
```



```
Enter option
Approx(58)          A:SEALPOP.MTH        Free:90%          Derive Algebra
```

STOCK.MTH

```
1:  "PROBLEM"

2:  "When I go into my local electrical goods store it always seems"

3:  "to be out of stock of the item I wish to buy. Could it be that"

4:  "it benefits the store to be regularly out of stock ?"

5:  "MODELLING ASSUMPTIONS"

6:  "Suppose the store sells portable compact disc players at £150."

7:  "The store buys these at £100 each with money which it borrows"

8:  "at a bank rate of 10% per year. It also pays £100 in transport costs"

9:  "for every delivery of new stock. Suppose finally that the store"

10: "can expect a constant demand for this item of 300 customers per year"

11: "and that every lost customer is lost profit !"

12: "MODEL"

13: "Let d be the stock reordering rate and p be the reordering period."

14: "If t is the time variable ,then :"

15: STOCK (t) := d p - 300 t

16: "This means that over the given period p the cost of the system is :"
```

17: $$\text{COST}(d, p) := 100 + 10 \int_{0}^{d\,p\,/\,300} \text{STOCK}(t)\,dt - 50 \int_{d\,p\,/\,300}^{p} \text{STOCK}(t)\,dt$$

18: "NOTE: use soLve on the following equation to understand the"

19: "integral limits in the expression for the cost."

20: STOCK (t) = 0

21: "Accordingly, the cost of the stock system per year is given by :"

22: $$\text{COST_PER_UNIT_TIME}(d, p) := \frac{\text{COST}(d, p)}{p}$$

23: "The objective is to minimise this COST_PER_UNIT_TIME."

24: "SETTING UP A WORKING SCREEN"

25: "Change to graphics display mode. Split window 1 horizontally ."

26: "Split window 2 vertically. Designate window 2 for 3-D Plot."

27: "Designate window 3 for 2-D Plot. Configure the windows as follows."

28: "Window 2: LENGTH: X:400,Y:2,Z:15000; CENTRE: X:300,Y:1,Z:8000;"

29: "EYE: X:1000,Y:-4,Z:30000; FOCAL: X:300;Y:1;Z:8000."

30: "Window 3: SCALE: X:0.2,Y:1000; MOVE: X:0.3,Y:660; CENTRE."

31: "ANALYSIS"

32: "Highlight and simplify the following expression"

33: $\text{COST_PER_UNIT_TIME}(d, p) := \dfrac{\text{COST}(d, p)}{p}$

34: "3D-Plot this simplified expression in Window 2. "

35: "d is represented by x ,p is represented by y and z represents"

36: "the COST_PER_UNIT_TIME in this 3D-Plot."

37: "Try altering CENTRE,EYE or FOCAL to gain a feel for this function"

38: "and a feel for where its optimal value might lie."

39: "An alternative approach is via a contour map. Simplify the "

40: "following expression which will generate a matrix of contour "

41: "functions for varying values of d and 2D-Plot this resulting"

42: "matrix in Window 3."

43: VECTOR (COST_PER_UNIT_TIME (d, p), d, 100, 500, 10)

44: "By moving the cross try to estimate the optimal value "

45: "of COST_PER_UNIT_TIME, and p. Can you work out an optimal value"

46: "for d ? [Hint: alter window 3 for a contour map with varying p.]"

47: "How many portable compact disc players should the store"

48: "regularly order ? And how often should it order them ?"

49: "Does it benefit the store to be regularly out of stock ?"

50: "ALTERNATIVE APPROACH"

51: "A traditional calculus based analysis can also be accomplished"

52: "with Derive.

53: "soLve the following equation for d and Manage Substitute"

54: "this solution into the second equation and soLve this for p."

55: $\frac{d}{dd}$ COST_PER_UNIT_TIME (d, p) = 0

56: $\frac{d}{dp}$ COST_PER_UNIT_TIME (d, p) = 0

57: "Check that these solutions correspond to your previous estimates."

58: "In particular, Manage Substitute your solutions into the following"

59: "expression and Simplify it to find the exact optimal cost per year."

60: COST_PER_UNIT_TIME (d, p)

61: "EXTENSIONS"

62: "Consider the effect of varying the STOCK function."

63: "Parameterise the modelling assumptions and use the calculus based'

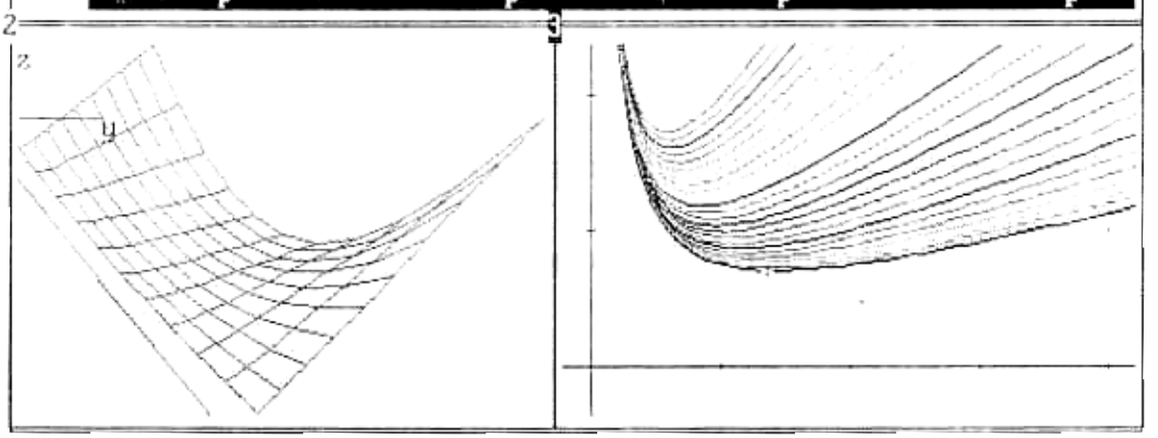
64: "analysis to develop a general stockout system theory."

Screen Layout for STOCK.MTH

64: "analysis to develop a general stockout system theory."

66: $$\frac{d^2 p^2 - 500 d^2 p + 75000 p^2 + 1000}{10 p}$$

67: $$\left[\frac{100 (35 p^2 + 1)}{p}, \frac{10 (321 p^2 + 10)}{p}; \frac{20 (147 p^2 + 5)}{p}, \frac{10 (269 p^2 +}{p}\right.$$



Enter option
Cross x:0.2722222 y:703.125 Scale x:0.2 y:1000 Derive 2D-plot

LOGISTIC.MTH

```
    μ := 1

2:  F (x) := μ x (1 - x)

3:  "F is known as the logistic function. Early interest was shown"

    "in this function as the basis of a population model by Verhulst (1838) in"

5:  "that it served to model the environmental constraint on population growth"

6:  "that was neglected by Malthus. However, the biologist Robert May"

7:  "noted in 1974 that as a population model it has some peculiar properties."

    "It is these peculiar dynamical properties that we shall investigate."

9:  "The following two functions ITERATION and COBWEB will help us to do this

10: ITERATION (n) := ITERATES ([ELEMENT (v, 1) + 1, F (ELEMENT (v, 2))], v, [1, 0.1 ê], n)

11: COBWEB (n) := ITERATES ([ELEMENT (v, 2), F (ELEMENT (v, 1))], v, [0.1 ê, 0.1 ê], n)

12: "As you will note ITERATION generates a matrix of coordinates"

13: "of the terms of the iteration against time. COBWEB generates a matrix"

14: "of coordinates too, but this time of the graphical analysis"

15: "of the iteration. By 2D-Plotting these coordinates we investigate  F."
```

16: "SETTING UP A WORKING SCREEN"

17: "Change to graphics display mode. Split window 1 horizontally."

18: "Split window 2 vertically. Designate both windows 2 and 3 for 2D-Plot."

19: "Configure the windows as follows."

20: "Window 2 : SCALE: X:20,Y:.3; MOVE: X:50,Y:.3; CENTRE."

21: "OPTIONS STATE MODE:Connected"

22: "Window 3 : SCALE: X:.5,Y:.5; MOVE: X:.5,Y:.5; CENTRE."

23: "OPTIONS STATE MODE:Connected"

24: "INVESTIGATION"

25: "Approx the following expression and 2D-Plot the result in Window 2."

26: ITERATION (100)

27: "2D-Plot the following expression in Window 3. You will note that"

28: "it produces a pair of graphs."

29: $\begin{bmatrix} F(x) \\ x \end{bmatrix}$

30: "Now Approx the following expression and 2D-Plot the result in Window 3."

31: COBWEB (100)

32: "[NOTE: by moving the cross in Window 3, pressing Centre and Zooming In"

33: "a clearer picture of the iteration can be obtained.]"

34: "The result is the graphical analysis of F. How are the two graphs "

35: "related ? The limiting value, called the attracting fixed point"

36: "is the solution to the following equation. Use soLve to find this point."

37: F (x)

38: "Repeat this investigation with μ:=2,2.5,2.9,3,3.2,3.5,3.56,3.57,3.6,4"

39: "[NOTE: all the useful expressions have been grouped together "

40: "after line 59 to help your work.]"

41: "Odd behaviour begins with μ=3. Can you explain it ?"

42: "For μ=3.2 superimpose the graph of the following function on Window 3"

43: F (F (x))

44: "Also soLve the following equation."

45: F (F (x)) = x

46: "And consider the values of the following at these solution points"

"by using Manage Substitute.

$$\frac{d}{dx} F(F(x))$$

49: "Robert May asked his graduate students what they thought might be"

50: "happening at μ=3.57. Can you suggest an idea ?"

51: "Other functions display the same behaviour as the logistic function."

52: "Feigenbaum investigated the following function in 1976"

53: $G(x) := \mu \, SIN(\pi x)$

54: "Consider also the following."

55: $H(x) := \mu (x^3 - 3x^2 + 2x)$

56: "What do these functions have in common besides similar behaviour ?"

57: "Can you suggest other functions that might have the same behaviour ?"

58: "Investigate your conjectures."

59: "THE FOLLOWING EXPRESSIONS HAVE BEEN GROUPED TOGETHER TO HELP YOU WORK"

60: $\mu := 1$

61: ITERATION (100)

62: $\begin{bmatrix} F(x) \\ x \end{bmatrix}$

63: COBWEB (100)

64: $F(x) = x$

65: $\frac{d}{dx} F(x)$

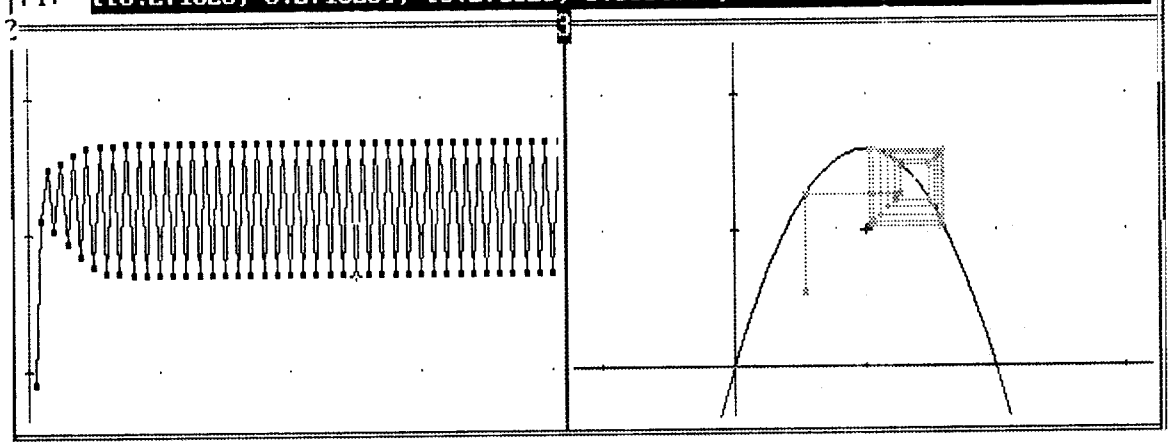
66: $F(F(x))$

67: $F(F(x)) = x$

68: $\frac{d}{dx} F(F(x))$

Screen Layout for LOGISTIC.MTH

```
1
69:  [[1, 0.271828], [2, 0.197937], [3, 0.158758], [4, 0.133554], [5, 0.115717
70:  [[0.271828, 0.271828], [0.271828, 0.197937], [0.197937, 0.197937], [0.197
71:  x = 0
72:  μ := 3.2
73:  [[1, 0.271828], [2, 0.633400], [3, 0.743053], [4, 0.610959], [5, 0.760601
74:  [[0.271828, 0.271828], [0.271828, 0.633400], [0.633400, 0.633400], [0.633
```



```
Enter option
Cross x:0.5        y:0.5        Scale x:0.5        y:0.5        Derive 2D-plot
```

MERSENNE.MTH

1: "In 1644 Mersenne stated that the number"

2: $M(n) := 2^n - 1$

3: "is prime for n = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257."

4: "Factor M (n) for these values in order to check this claim"

5: "NOTE: It took the American mathematician Frederick Cole 20 years to"

6: "find the prime factorisation of M (67) in 1903."

7: "How long does it take DERIVE ?"

8: "Can you find any values of n 50 < n < 100 which Mersenne has missed ?"

9: "WHEN YOU'VE FINISHED TRANSFER LOAD FERMAT1.MTH"

FERMAT1.MTH

```
1:  "In a letter to Mersenne in 1640 Fermat wrote"

2:  "'I have found that numbers of the form"

                 n
               2
    F (n) := 2   + 1

4:  "'are always prime numbers and have long signified to analysts the"

5:  "truth of this theorem'."

6:  "Use Factor to investigate this claim."

7:  "WHEN YOU'VE FINISHED TRANSFER LOAD UTILITY MISC.MTH"

8:  "AND TRANSFER LOAD FERMAT2.MTH"
```

FERMAT2.MTH

```
1:  "TRANSFER LOAD UTILITY MISC.MTH BEFORE LOADING THIS FILE"

2:  "From MERSENNE and FERMAT1 you may have noticed how quickly DERIVE"

3:  "recognises a prime number as against how long it takes for DERIVE"

4:  "to prime factorise a composite number. A test for primality"

5:  "exists independently of trial division which is based on Fermat's little"

6:  "theorem."

                          n - 1
7:  PPRIME (n, a) := MOD (a      - 1, n)

8:  "Fermat's theorem states that PPRIME(n,a) = 0 if n is prime "

9:  "and n,a are coprime."

10: "Check the following."

11: PPRIME (7, 3)

12: PPRIME (11, 2)

13: PPRIME (23, 2)

14: PPRIME (61, 2)

15: "However, consider:"

16: PPRIME (341, 2)
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17: "Is 341 prime ?"

18: "Check"

19: PPRIME (341, 3)

20: "Consider for any integer a coprime to 561"

21: PPRIME (561, a)

22: "Any composite n such that PPRIME (n,a) = 0 is called a pseudoprime to"

23: "base a. There are 14884 psuedoprimes to base 2 less than " 10^{10}

24: "There are 455052512 primes less than " 10^{10}

25: "Any composite n such that PPRIME (n,a) = 0 for all coprime bases a"

26: "is called a Carmichael number. They are very rare.

27: "Confirm that 6601 is a Carmichael number."

28: $M(n) := 2^{n} - 1$

29: $F(n) := 2^{2^{n}} + 1$

30: "Every composite Mersenne number M (n) and every composite Fermat"

31: "number F (n) is pseudoprime to base 2. Can you show this ?"

32: "Is this what misled Mersenne and Fermat ?"

33: STRONG_PPRIME (n, a) := MOD $(a^{(n-1)/2}, n)$

34: "If n is prime STRONG_PPRIME(n,a) = 1 or n-1. (Miller)"

35: "Check this theorem and also try the following."

36: STRONG_PPRIME (561, 5)

37: "Strong pseudoprimes are extremely rare. Consider:"

38: STRONG_PPRIME (2047, 2)

39: "The only strong pseudoprime less than " $25\ 10^{9}$

40: "to bases 2, 3, 5 and 7 is 3215031751. However, DERIVE has its limitations."

41: "Simplify the following to see this !"

42: STRONG_PPRIME (3215031751, 2)

43: "Suggest a reasonable way of testing for primality which does not involve"

44: "trial division."