BACK

DERIVE in the Dutch Classroom

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Introduction

In the Netherlands, just as in other countries, the development of powerful, user-friendly and cheap Computer Algebra Systems (CAS) like Derive leads to serious questions about mathematics education on different levels. What will be the consequences for didactics and curriculum? How are these tools to be incorporated in the teaching and learning of mathematics?
At the Freudenthal Institute, a research group on mathematics and computer science education, started a project called 'Computer algebra at upper secondary level' in August 1990.
In this paper we present some results from this project. In the next section we will describe the main objectives and activities of this project. Then you will find two examples taken from the student textbook that will be produced as a result of the project. A section with some conclusions follows, and we conclude with some recommendations for the further development of Derive.

The project

In this project we concentrate on upper secondary level, i.e. students of age 16-18. Aim of the project is to develop a sound attitude towards the use of computer algebra in mathematics education on this level, and more specifically to foresee the consequences for the didactics, the curriculum and assessment of mathematics in secondary education.

Here are some research questions:

* How can the use of a CAS lead to an improvement of the teaching and learning of mathematics? How can we incorporate computer algebra in mathematics education in a didactic way? What consequences will there be for the didactics? For example, is it possible to focus on the underlying mathematical concepts rather than on algorithmic skills?
* What changes in the mathematics curriculum will take place due to computer algebra?
* Can the use of a CAS support the philosophy of realistic mathematics education? For example, can we focus more on the modeling aspects of a realistic situation than before, because our tools free us from calculational problems?

It is too early to answer these questions. In order to gather information and experience, our main activities so far have been:

* The development of student texts for the learning of mathematics using Derive. A booklet containing a collection of these hand-outs will be published this summer. We consider this an important result of the project, not in the least because it can serve as an example for teachers and it may stimulate them to think about their lessons in the future and to develop their own student texts.
* The classroom experiments form another important part of our project. At different schools and on different levels teachers use our labs with their students. During the year 1991-1992 about 12 schools have been participating. Of course, the results of these experiments are extremely important to us. In fact, this is the way in which we hope to find answers to the basic questions above: an interaction between the development of student materials, classroom experiments and theoretical reflections and research.
* The development of refresher-courses for mathematics teachers and courses for future mathematics teachers. The contacts with teachers attending these courses are important sources of feedback for us.

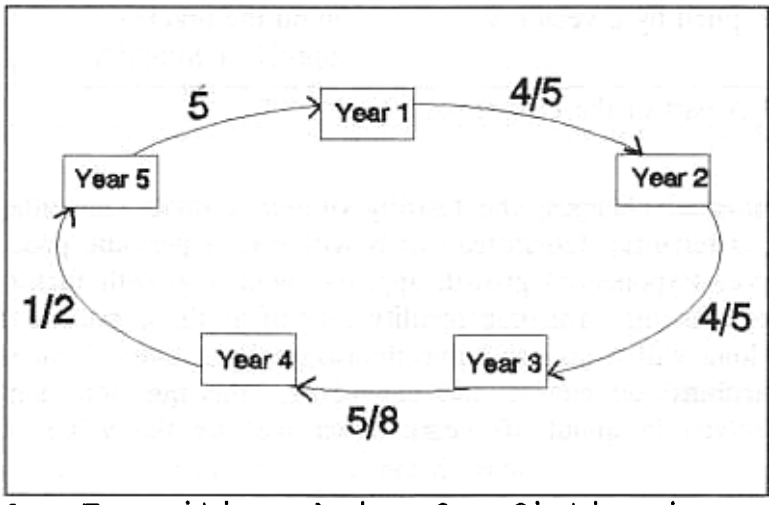
In the next two sections you will find examples from the student materials. Let me tell you how these examples are presented. In the teacher version of the textbook, each Derive lab is described in the following way.

1. Teacher page(s), containing the aims of the session, a global description, preliminary mathematical knowledge, required Derive skills, and practical suggestions for the use of this text in the classroom.
2. Task page(s), containing the mathematical problems for the student. We do not want our Derive-lessons to become button-pressing lessons, so we prefer to seperate the mathematical contents from technical details. The students primary concern is the mathematical problem that has to be solved, and it is up to him or her to use paper and pen, Derive or the knowledge of his neighbour. Of course, the problems are of a kind that suggests the use of Derive.
3. Derive page, containing instructions for the use of Derive for this specific problem. The student has to choose himself which is an appropriate suggestion to find an answer to his question at that moment. This forces him to realise what he really needs to know, in other words to be conscious of his problem-solving strategy.

4 A general Derive Help page is always present. It contains information about the handling of Derive, for example: use Alt-P for π, use Esc to go back to the main Algebra menu, and so on.

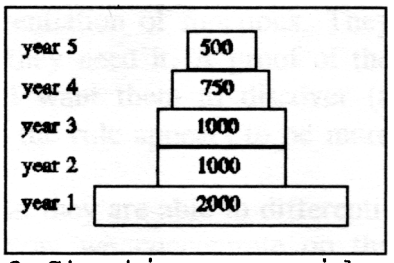
First example: a population matrix

Let me first tell you something about mathematics education in the Netherlands. On the upper secondary level, we teach the so-called Mathematics A and Mathematics B. Mathematics A is oriented towards the future student in economics, psychology, medicine and so on. It contains statistics, matrix algebra and calculus, but the latter in a rather intuitive, less formal way. Modeling and the applications of mathematics to real world situations play an important role in Mathematics A. Mathematics B is for the student who is interested in physics, chemistry and so on. It contains geometry as well as the more formal approach to calculus. Students can decide to choose Mathematics A or B or even both or none. The example that is described below fits in the Mathematics A curriculum.
Operations on matrices are quite tedious. Miscalculations are easily made, while difficult concepts are not involved. Rather than to stress the importance of hand calculation, it might be interesting to focus on the modeling aspect of migration matrices and on the interpretation of the results. So let us look at figure 1.



1 Transition chain of a fictive insect species

Suppose insects of a fictive species can live 5 years. In their last year, they produce eggs that will give an average of 5 young insects the next year. In figure 1, you also see the survival chances. For example, 5/8 of the insects in their third year will survive the next winter. The population we start with is presented in the pyramid of figure 2.



2 Starting pyramid

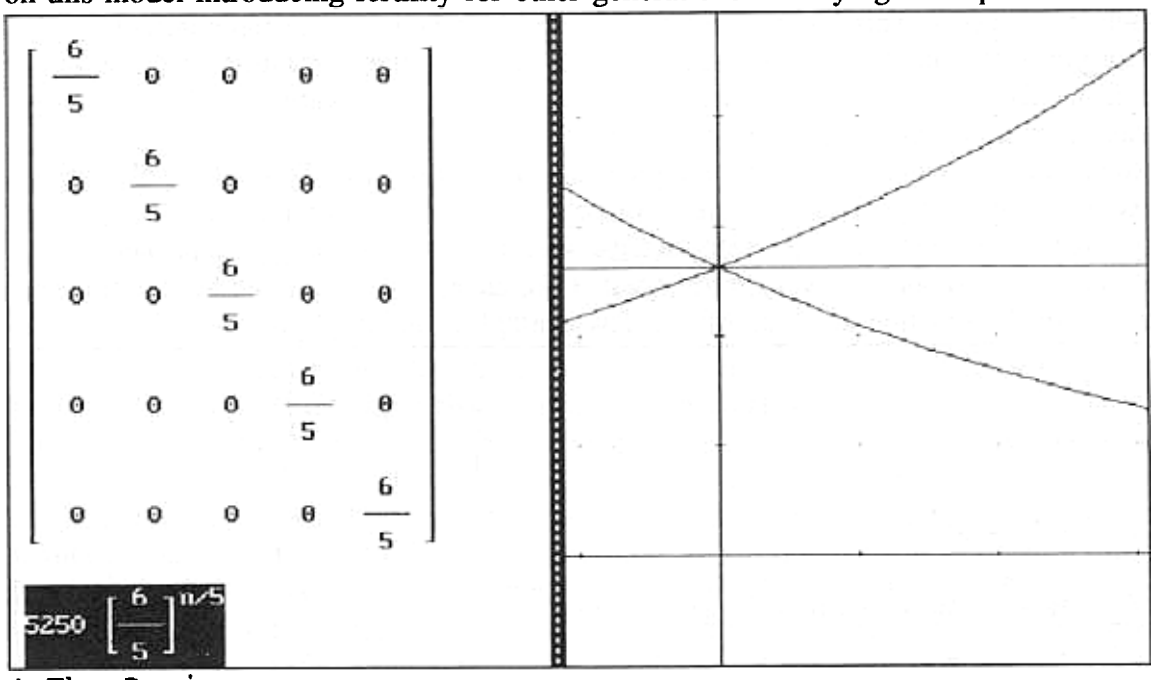
The first problem for students is to find the migration matrix. Then they can calculate the population pyramid in future years. They find out that after 5 years, this pyramid equals the original starting situation, so there is a cycloïd process going on. Once you notice this, you can easily check this by just following one 'generation' during 5 years or by verifying that the fifth power of the matrix is the identity matrix. So after 5 years, the process is repeated. Can you find out a partition of the 5250 insects that remains the same year by year?
To give you an impression, you see in figure 3 a part of the Derive page with some relevant information for the use of Derive in this context.

What do you want?	How can you do it?
multiply a matrix M with a vector V	Author then type: M.V approX or Simplify
calculate the nth power of a matrix M	Author then type: M^n approX or Simplify
calculate the nth power of a matrix M multiplied by a vector V	Author then type: (M^n).V (mind the brackets!) approX or Simplify

3 A part of the Derive page

Due to environmental changes, the fertility of our animals can either increase or decrease. E.g., a fertility of 6 instead of 5 will give a periodic process around an exponential curve. Exponential growth appears, with a growth factor of 6/5 for a period of 5 years. In the case of a fertility rate of 4, the exponential curve is decreasing. How long will it take to halve the original population? An introduction to the use of logarithms! Of course, one can reverse this question. Suppose that the population is halved in about 10 years, what will be the value of the fertility constant?

As you see in figure 4 we can conclude that this model leads to an equilibrium that is sensitive to changes in the fertility constant. Of course, one can continue the work on this model introducing fertility for other generations or varying other parameters.



4 The Derive screen

Some final remarks about this example:

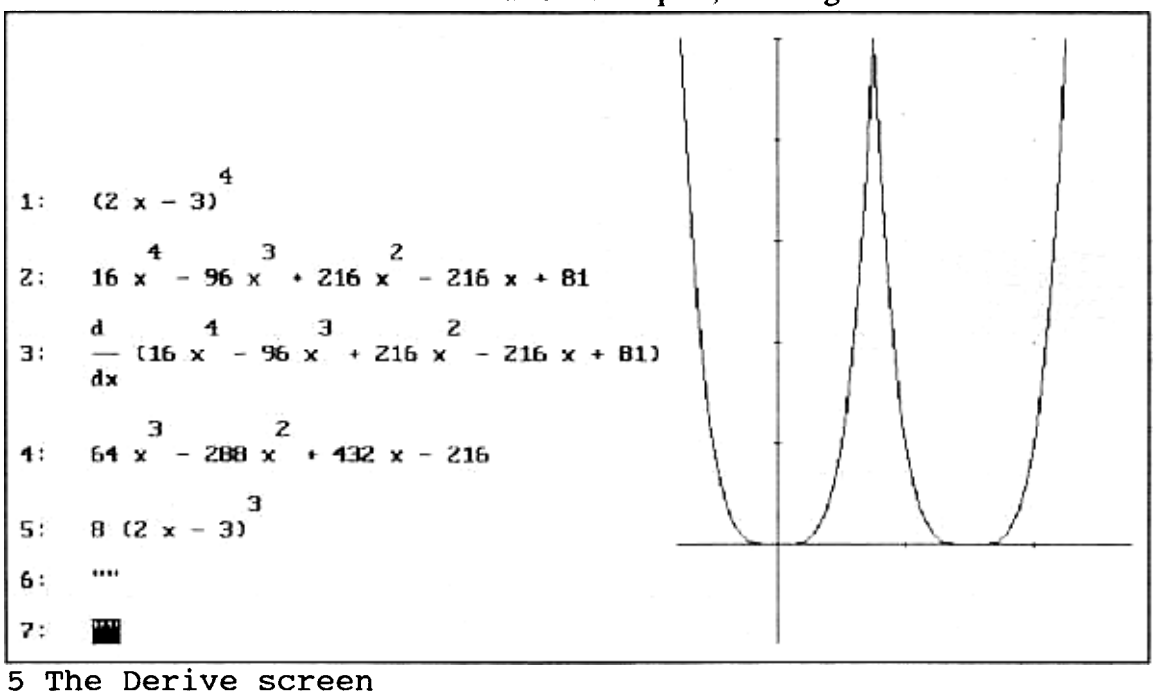
* The ability to perform matrix calculations by hand is irrelevant when you use Derive.
* Modeling aspects become more important. One has to be able to set up the matrix and interpret the results. One can change parameters and study the effects.
* The use of Derive supports the link between different subjects, in this case migration matrices and exponential growth.
* The graphing of functions forces the student to reflect on domain and range.
* One can easily experiment, conjecture and verify when the calculational drudgery is left to a Computer Algebra System.

Second example: anticipating the chain rule

Students often do not like the Chain Rule for the differentiation of functions. They tend to forget to use it and they do not really see why they need it. A proof of the theorem does not help because it is too abstract. So I want them to discover (a simple case of) the Chain Rule themselves. In this way, the rule appears to be more natural and real.

Suppose our students know what differentiation is and that they are able to differentiate expressions such as $x^3-6x^2+37x-101$ and so on. Now we concentrate on the derivative of functions of the class $f(x)=(ax+b)^n$. In itself, this is not an extremely important class of functions but the advantage of this example is that there are two

ways to differentiate the expressions without using the Chain Rule.
Let us first work out a simple example by hand: $f(x)=(x-1)^2$. To differentiate this function we expand to x^2-2x+1, which the students can differentiate to $2x-2$, which factors to $2(x-1)$. This looks like a familiar rule: in fact, you differentiate $(x-1)^2$ the way you differentiate x^2. A second approach might be the translation of the graph 1 unit to the left. The new function is $g(x)=x^2$, new derivative is $2x$ which we translate back to the original derivative $2(x-1)$.
Now, students can repeat differentiating functions this way. Soon, the calculations become more complex, and Derive can help. See figure 5 for the Derive screen of an excellent student. Please, note that the student can verify every step! The use of Derive at this moment is very transparent. Of course, a short cut can be the differentiation of the function at once. After a few examples, one might do so.



5 The Derive screen

Quite soon, students begin to conjecture about the general form of the derivative. They can verify this by choosing their own values of a, b and n. A complication may show up: Derive simplifies the derivative of $(2x-6)^5$ to $160(x-3)^4$. Recently, I attended an interesting discussion among students about the question: Is the derivative of $(ax+b)^n$ $n.a.(ax+b)^{n-1}$ or is it $n.a^n.(x+b/a)^{n-1}$?
A lot of generalisations are possible from this starting point. Is the rule you discovered true for every value of a, b and n? Can you find the derivative of $f(x)=(ax^2+bx+c)^n$? And how about the derivative of $\sin(ax+b)$?
Some final remarks about this example:

* A Computer Algebra System can serve as a generator of examples that permit the user to experiment and to train pattern recognition.
* Students (like myself) like working with examples that are larger than the ones you use when you do everything by hand. For example, to expand $(8x-17)^9$ by just pressing two buttons is amusing and impressing!
* The use of CAS leads to other approaches of the 'classical' themes, and it can also facilitate the use of more than one approach.

Student reactions and conclusions

In general, students are enthusiastic about their work with Derive. They are impressed by the enormous power. They quickly start to experiment, sometimes with beautiful results. And soon you can hear questions such as: why do we have to do all this stuff by hand?

In my opinion, teaching how to use a CAS is an objective in itself for mathematics education. Furthermore, in the examples given above we have seen important advantages of the use of compteralgebra: no calculational drudgery, training of problem solving and pattern recognition skills, different ways of working on a problem, a focus on modeling or on choosing a strategy or on the relevant concept rather than on 'dull' algorithms.

From the experiences in the classroom, however, I would like to give some practical suggestions and point out some risks that are worth mentioning.

For students, it is important that a Derive session takes place at exactly the right time. If a problem is not relevant in the context of the preceding lessons, they will not be motivated. Also, the mathematical notation and language used in the Derive session should fit perfectly in their 'tradition'. If it doesn't you will lose time translating. Furthermore, it is important to point out to the students that the Derive lesson is not just fun, but that they have to learn something from it. You can stimulate them to work seriously by evaluating their written reports.

The students generally have a hard job to do during Derive lessons: they have to think about mathematical problems and at the same time about the handling of Derive, a programme that is often not really familiar to them. So they focus on the PC, and little energy is left for reflections. They do not lean back to say: let us see what we are doing and why. So here we have an important task for the teacher: to stimulate the students to reflect, asking them questions such as 'What are you doing?', "Why do you do it?', 'What are the conclusions so far?', 'What will be the next step?', 'Can you foresee the result of the next step?' and so on.

To make it more easy for students to recognize the theme in the Derive session, it is important for the teacher to introduce the session, to work out an example (by hand?) and to reactivate some preliminary knowledge before the students start to work on their own. After the session, an overview by the teacher will be of great value.

You see that the teacher has a tough job as well: a teaching task, a Derive trouble shooting task, an organisation task.....

There is a last point that may be important. A CAS is a powerful tool. For students it may be too powerful. There are many possibilities, and the techniques that are used as well as the answers that are produced are often far beyond the students' knowledge. Take for example the use of complex numbers that students encounter when they try to solve simple quadratic equations.

Teachers on the other hand are fascinated by the power of the programme. So they tend to jump to abstractions that are made too fast while the handwork phase of simple examples is skipped. Students will not understand what they are doing and miss the experience to follow the abstraction. This will make mathematics very hard to do for them. In my opinion this is the main risk of the use of computer algebra: neglect of the handwork phase and abstraction made too fast and to a level that is too high.

The Derive user interface

Suppose that during a certain lesson you have all your students working with Derive in a PC lab. There are two simple questions to answer in order to know if it is a good lesson. First question: are you, the teacher, busy? If the answer is no, congratulations! If the answer is yes, the second question becomes relevant: what are you busy with? If the answer is: discussing the mathematical problem with students, asking students why they calculate what they calculate, stimulate students to formulate the conclusions so far, asking students what experiments would now be useful, and so on: congratulations again! But if you run from PC to PC to explain the students what you want them to do, and what it has to do with the mathematics lessons, or if you are a trouble shooter who constantly has to close windows in a screen that has ten of them, then you have a problem.

There are a lot of factors that create these problems some of which are due to interface problems of Derive. Although students learn to use Derive very fast, and although Derive is quite user-friendly, there are some points that are, at least in my experience, annoying.

Take the first example about migration matrices. Students have to change the matrix, but they have two problems which they rarely solve without help: the old Wordstar editing keys, and the line representation of a matrix. Why can't we activate an edit-window with F1 and then use the normal arrow keys to edit? Why can't we enlarge the edit window?

In the second example about the chain rule, I want the students to work with a split-screen with an algebra window and a plotting window. In Derive version 2.01, they have a lot of keystrokes to make to achieve this. I am glad to know that version 2.10 will solve this problem in the future!

As a final remark I'd like to stress the importance of the support of the use of a mouse. This might facilitate the use of Derive. One no longer has to know that the Tab key takes you from the number of rows of a matrix to the number of columns and other details.