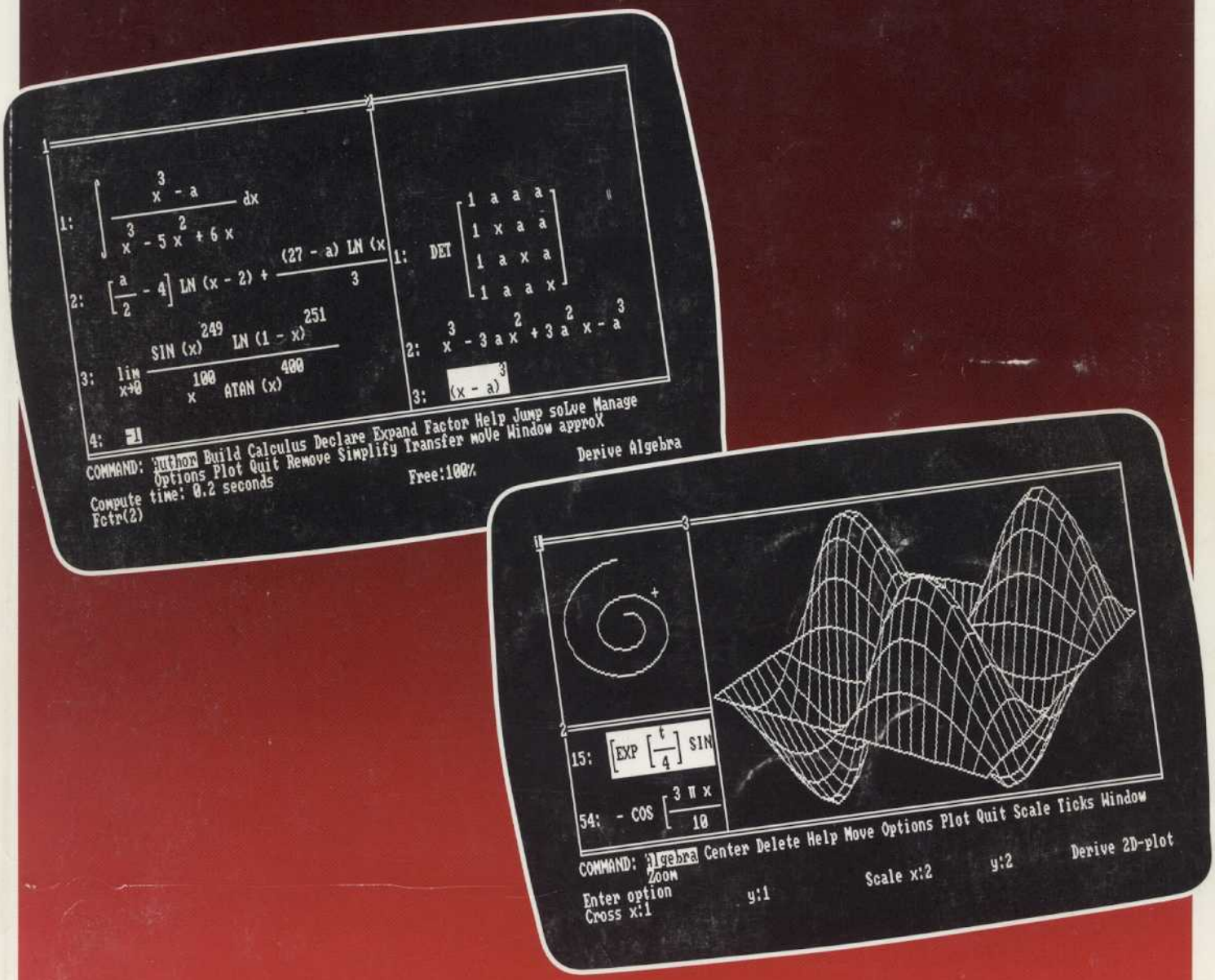
Edited by Josef Böhm

Teaching Mathematics with DERIVE®



Chartwell Bratt

Teaching Mathematics with *DERIVE*®

Proceedings of the
International School on the
Didactics of Computer Algebra
(April 27-30, 1992, Krems, Austria)

Josef Böhm, Editor

Chartwell Bratt

Dedicated to Dennis Tanner.

Dr. Dennis Alan Tanner (13 March 1945 - 5 May 1992)

At a time when developments in symbolic manipulators and computer aided learning are presenting exciting new challenges, it is a tragedy that one who had so much to offer in energy, imagination and drive cannot now meet those challenges.

After a period of consolidating his scholarship, Dennis blossomed in 1985 when he joined a small group of colleagues in the Mathematics Department at City University in devising innovative experimental laboratories for mathematics students. In these early days, teaching software was written for the intelligent, but limited, BBC Econet. Soon, it became apparent to Dennis that further development would not be possible without redesigning the system to use the powerful software demanded by mathematicians and statisticians. He took the lead in building a computational mathematics laboratory based on PCs which would run symbolic manipulators, graphics, statistics packages, spreadsheets, word processors and numerical packages. So that the system would be transparent to the student, he wrote system software and developed a robust shell. His knowledge and expertise was valued by students and staff alike. As testament to his endeavours, City University now has enviable facilities for computational mathematics in the UK.

In the past two years, Dennis had realised that *DERIVE* provided an opportunity for further innovation in teaching methods and he became a *DERIVE* consultant in 1991. He was active in the affairs of the Computers in Teaching Initiative at Birmingham University and he advised institutions on the setting up of computational mathematics laboratories. Dennis was a principal participant in a new ERASMUS programme in Germany.

Dennis was not purely a PC man: he could work equally effectively in a Unix environment. He contributed to wider issues on the Information Technology Policy Committee in the University, where his no-nonsense attitude and direct manner often had an incisive effect on the decisions made. His academic post was as a lecturer in mathematics, but he also had heavy responsibilities as Computer Manager for the School of Mathematics, Actuarial Science and Statistics.

Dennis was born on 13 March 1945 in Bristol and retained his love for the West Country all his life. He went to Fairfield Grammar School and gained his BSc in Applied Mathematics from Reading University in 1966. He was a scientific officer at the National Physical Laboratory, Teddington for five years from October 1966, during which time he completed his Reading University PhD in 1971 on Cnoidal Waves. He joined City University as a lecturer in mathematics in November 1971, forging working friendships with colleagues over a period of 21 years, presenting a charismatic approach to students of several generations and rather enjoying his later persona as "Hi-Tech Den".

During his early career at City, he met Sheena, whom he loved dearly and who gave him so much throughout their marriage to the very end. Our hearts go to Sheena and to his mother in our vivid memories of a very dear friend.

Chris Haines
27 May, 1992

Foreword by the Representative of the Austrian Ministry of Education

When man created mathematics, he also invented tools for doing mathematics more easily, and these tools also influenced the teaching of mathematics. In the sixties of this century it was the calculator, in the seventies a new challenge arose: the computer.

So we began research in this field, and we tried to change the teaching of mathematics. We hypothesised that learning of informatics will enable pupils to get better scores in mathematics. But this hypothesis could not be proven. On the contrary: we found out that the results of the better pupils grew better, the results of the others grow worse. So we stopped our experiment and created a new subject: informatics.

At first this subject was optional. Since 1985 it has been compulsory for one year in the ninth grade and optional for higher grades. Since 1989 three to four subjects have included computer literacy at the lower secondary level: German, English, Mathematics and Descriptive Geometry. For this purpose each Austrian general secondary school has at least one local area network with fourteen computers.

In mathematics mostly spreadsheets are used. But spreadsheets are only of limited use in the teaching of mathematics. So some teachers tried to use *muMATH*. In 1991 the Austrian Ministry of Education and the Arts decided to buy *DERIVE* for all general secondary schools. Naturally it was not used everywhere.

So what we need are pilot projects for working with *DERIVE* in the classroom and extensive feedback and discussion between all those who are working in these projects. Even more we need communication among all those working in this field, such as experts on the didactics of mathematics and experts on software engineering; and not only at a national level, but also together with experts from different countries. That was the reason for organising a seminar on the didactics of computer algebra in April 1992.

I am sure, it has been a useful and interesting seminar. We had a fruitful exchange of ideas and experiences in teaching mathematics and other subjects with computer algebra software. I hope that the proceedings of this seminar will stimulate discussion in this field.

Eduard Szirucsek
21 September, 1992

Foreword by the Conference Chairman

The invention of tools always has had a direct influence on the development and didactics of mathematics. The recent introduction of computers into the classroom will have a tremendous influence on the theoretical and practical methodology of mathematics and mathematics education.

During the 70s, when numerical pocket calculators became widely available, teachers, schools, and school authorities quickly accepted them as a tool to take off the burden of extensive numerical computations; like the multiplication of two five digit numbers or the evaluation of a trig function. Numerical calculators were readily embedded in the overall process of math teaching.

Now, in the 90s, "symbolic calculators", so called computer algebra systems, become more and more popular. They have the power to automate the drudgery of symbolic manipulations like simplification of an expression or computation of the antiderivative of a function. Their full impact on mathematics in general and math teaching in particular will be tremendous and cannot be even estimated now.

Those, who have worked with computer algebra systems before, are convinced that they will drastically change our didactics of mathematics. Like every tool, computer algebra can do good and bad. And we have to find out how to use it to do good. Live experiments at schools are the only instrument to find out, how computer algebra systems can be used to *improve* math teaching.

Tu Felix Austria, because the Austrian Ministry of Education has equipped our grammar schools with all means that are necessary to employ computer algebra on a very broad basis: Every school has 14 PCs which are linked in a network. Last year, the Ministry purchased a nationwide license of the computer algebra system *DERIVE*. The new curriculum requests that teachers make use of computers in as many subjects as possible from grade level seven on. Austria is the first country in the world that has recognized the importance of computer algebra for teaching mathematics.

However, it does not suffice to supply teachers with disks and user's manuals. Teachers need didactic guidance also. This requires that we ask not only "WHAT", but also "HOW". In developing a didactics of computer algebra we have to distinguish two different situations:
(1) Students having access to computer algebra systems only on the school computers. (This requires that math classes will be held in the computer labs.)
(2) Students having available computer algebra systems on portable computers (laptop or palmtop computers) and using them in all phases of learning: at school during regular classes, at home doing their assignments, and at school working on the exams.
It seems clear that the second situation is what we all want. Recent developments on the hardware market make me very optimistic.

The goals of our conference were
- to compile material for teaching mathematics with computer algebra,
- to develop guidelines and concepts for teacher training,
- to discuss relevant aspects of the curriculum, and
- to stimulate the further improvement of the software.

Personally, I regard this symposium as an important milestone. In times of increasing international cooperation, the exchange of expertise and experience should be a logical consequence. It would be silly to "reinvent the wheel" in every single country.

We have chosen the idyllic town of Krems as the appropriate venue for this symposium not only because we believe that a pleasant and inspiring atmosphere contributes towards the success of such an event, but also, because the "Landesakademie" is intended to become a focus of future scientific and academic activities in the State of Lower Austria.

I express my sincere thanks to the Austrian Ministry of Education for funding this conference. A special "Thank You" goes to Ministerialrat Dr. Eduard Szirucsek, who is the representative of the Ministry of Education. His personal interest and expertise in the subject of this meeting has been a motor to all related activities in Austria. Finally, I thank the executive body of the "Landesakademie" for their warm welcome and for their support in organising the conference.

Helmut Heugl
15 September, 1992

Foreword by the Editor

Computer algebra is a new technology with an immense impact on technical sciences. Computer algebra systems become more and more widely available, even on simple and affordable hardware. How should math teaching be changed to cope with this new development?

The "International Spring School on the Didactics of Computer Algebra", held April 27-30, 1992 in Krems, Austria, aimed at bringing together people with an interest in developing a didactics of computer algebra for the math education of students aged 12-18. The program of the meeting consisted of presentations of classroom experiments, discussions, and working groups.

As a member of the program committee and president of the *DERIVE* User Group I took the responsibility to edit this proceedings.

I thank the organizers of the conference for all their efforts. Krems'92 has had a very special spirit - which none of us will ever forget. We met Sunday evening as strangers, as people with a common interest in a certain subject, and parted on Wednesday evening as friends. I am convinced that the meeting was a milestone in the effort to revolutionize the teaching of mathematics.

Thanks to all lecturers for their cooperation in editing this proceedings. And thanks to Philip Yorke from Chartwell-Bratt for his efforts and cooperation in publishing them.

Josef Böhm
15 September, 1992

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