Computer and Education: A High-School Experiment Using the Mathematical Software *DERIVE*

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Abstract

This project involves the study of math in a Brazilian high-school using Derive and LOGO methodology, which allows students to explore their thinking process and many subject areas such as: matrices and functions.

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1 - Introduction

Today the world depends on technology. We can not imagine it without computers anymore. So, schools will be different: they will need such technology for improving themselves.

Schools have to modernize their structure, otherwise they will lose their students. At the present time we are living a period with the challenge of educational technology. Thus computers became the main tool to motivate students in order to explore subjects instead of sitting at their desks listening to the teachers talking about things.

Nevertheless, Brasil is a third world country and does not have enough money to invest in this "school of the future". We have to improve our students. They need more and they want more. Thus considering such shortage of money we have to choose a defined pedagogical computer strategy.

Our point of view is that Derive [1][2] means the best solution for a third world country because it is compact, cheap and runs in a simple PCXt. It is the best way to introduce technology in math studies at Brazilian schools. We are developing a kind of Derive Project in our University. workbooks and list of exercises are being organized. In this way we are intending to follow the Risc experience. Therefore this contact is fundamental for us.

2 - The Project

Objective: Motivate students for studying mathematics. Show mathematics as a dynamic thing.

There was detected a problem with mathematics in a high-school class at Colégio de Aplicação, Universidade Católica de Petrópolis. Computers appear as a teaching

aid using software. Derive, because of its viability and for being user friendly. This project is based on cognitive ideas from Jean-Piaget [3][4] and Seymour Papert's methodology - LOGO [5].

It intends to investigate how mathematical software can be used as an educational tool and its ability to change mathematical teaching, both in the methodological and technical aspects. Educators have to know that time does not stop and they have a compromise to improve learning, for our future and survival as a society. Education needs help and computers can help, but if educators think that a machine can solve all the problems, they are completely wrong. Methodology is very important. A computer is only a machine and software only a program. Teachers and students could lose themselves without a methodology. Methodology gives the ways. Logo methodology is a Seymour Papect approach for education. Much more than a turtle, it is there to teach thinking about all the process and help the teachers to understand students' way of thinking.

You can teach everything to everyone in their level of understandability. Students have their theories about subjects and teachers have to respect it. The process of thinking is wonderful. Knowledge is building step by step and at the end you will be able to use this, you have been building your "base of knowledge". Well, it is time to apply and improve it. Students like to explore new things, so let them do it . Derive permits a good trip throughout the knowledge. It can really help students. Much more than just math software, Derive can be used to explore concepts and help students to understand what is really happening. When students are working with Derive they are learning a language to speak about form, process and experience. Using Derive as a tool for teaching, students begin to think about why they are doing such things and why these things happen. It is a process of learning. They experiment, commit mistakes, give answers and at the end of the process they are convinced that they are learning things. It is interesting to observe that when students are trying to solve a problem, they start to think how and why Derive gave such answers. How and why are magic words for knowledge.

Unfortunately, schools teach numbers, rules, equations, but do not teach the

why of that. The knowledge is disassociated of the context, so it does not make sense for students. It is boring and difficult. If you do not understand the beginning, you will never understand it properly. You will not like what you do rather than dominate it. Derive shows that math is not a distant thing (that it can be a pleasure instead of hell). Derive helps to make hypotheses and stimulate creativity. When they find the correct answer, they will be happy and self-confident: "I did it"

This is the goal: compare + think + ask + answer = comprehend.

3 - The Experiment

It is expected that students motivate themselves to explore the arising software problems to discuss and transform math as an amusing subject. As a correlated hypothesis it is expected that it could be possible to save students from boring activities (such as a mechanized process for calculus without contextual understanding); show why students must learn such things (i.e. where that formula comes from and where it should be); expose math's importance for life; show new horizons for mathematical educators; detect students' problems faster; teachers must participate as educators, not instructors. First, it was given a trigonometry emphasis, because of the lack of understanding. Exercises were made first in a workbook and later in Derive. It is very interesting to observe what happened when students did their exercises in a workbook and after in Derive.

These exercises are from students' book [6].

1) In a Right-angled triangle, the legs are b = 15 cm and c = 20 cm. Determine sin (c), cos (c) and tan (c).

WORKBOOK

$b = 15$

$c = 20$

$a^2 = b^2 + c^2$

$a^2 = 15^2 + 20^2$

$a^2 = 225 + 400$

$a^2 = 625$

$a = \sqrt{625}$

$a = 25$ MUCH MORE STEPS TO FIND AN ATTRIBUTE

$\sin(c) = \frac{c}{a}$

$\sin(c) = \frac{20}{25}$

$\sin(c) = \frac{4}{5}$

$\cos(c) = \frac{b}{a}$

$\cos(c) = \frac{15}{25}$

$\cos(c) = \frac{3}{5}$

$tg(c) = \frac{c}{b}$

$tg(c) = \frac{4}{3}$

16 STEPS

DERIVE

1: $b := 15$ ATTRIBUTES

$2: \; c := 20$

$3: \; a = \mathrm{sqrt}\left(b^2 + c^2\right)$ POSITIVE PART, STUDENTS THINK MUCH MORE

$4: \; a = 25$ SIMPLIFYING ...

$5: \; \sin(x) = \frac{c}{a}$

$6: \; x = \mathrm{asin}\frac{20}{a}\mathrm{sign}(a)$ SOLVE DOESN'T WORK!

$7: \; \sin(x) = \frac{20}{a}$ BUT SIMPLIFY ...

$8: \; a := 25$ OOPS! I FORGOT TO ATTRIBUTE a

$9: \; \sin(x) = \frac{4}{5}$

$10: \; \cos(x) = \frac{b}{a}$

$11: \; \cos(x) = \frac{3}{5}$

$12: \; \tan(x) = \frac{c}{b}$

$13: \; \tan(x) = \frac{4}{3}$

13 STEPS

2) In a right-angled triangle, $b = 4$cm and $a - c = 2$. b and c are legs. Find $\tan(\gamma)$.

WORKBOOK

$b = 4$

$a - c = 2$

$a^2 = b^2 + c^2$

So,

$a = c + 2$

Substitute:

$(c+2)^2 = b^2 + c^2$

$(c+2)^2 = 4^2 + c^2$

$c^2 + 4c + 4 = 16 + c^2$

$4c + 4 = 16$

$4c = 16 - 4$

$c = \frac{12}{4}$

$c = 3$

$\tan(\gamma) = \frac{c}{b}$ THE ANSWERS

$\tan(\gamma) = \frac{3}{4}$ 13 STEPS

DERIVE

1: $b := 4$ ATTRIBUTES

2: $a := c + 2$

3: $c + 2 = \text{sqrt}\left(b^2 + c^2\right)$ LIKE $a = \sqrt{b^2 + c^2}$ SIMPLIFYING

4: $c + 2 + \text{sqrt}\left(c^2 + 16\right)$ OOPS! SOLVE!

$5: \ c = 3$ ATTRIBUTE

$6: \ c := 3$

$7: \ \tan(x) = \frac{c}{b}$ THE ANSWERS!!!

8: $\tan(x) = \frac{3}{4}$ 8 STEPS

3) In a right angled triangle the hypotenuse a=25. cos(b)=0.96.

Determine the triangle perimeter.

WORKBOOK

$a = 25$

$\cos(\beta) = 0.96$

$\cos(\beta) = \frac{c}{a}$

$0.96 = \frac{c}{25}$

$c = 0.96 \cdot 25$

$c = 24$

$a^2 = b^2 + c^2$

$25^2 = b^2 + 24^2$

$b = \sqrt{25^2 - 24^2}$

$b = \sqrt{625 - 576}$

$b = \sqrt{49}$

$b = 7$

$2p = a + b + c$

$2p = 56$ 14 STEPS

DERIVE

1: $a := 25$ ATTRIBUTES

2: $\cos(\beta) = 0.96$

3: $0.96 = \frac{c}{a}$

4: $c = 24$ ATTRIBUTES

5: c:=24

6: $a = \text{sqrt}\left(b^2 + c^2\right)$

7: $b = 7$ OOPS! TWO ANSWERS!!

8: $b = -7$

9: $b = \text{sqrt}\left(a^2 - c^2\right)$

10: $b = 7$ MUCH BETTER!

11: $b := 7$

12: $a + b + c$ PERIMETER

13: 56 THE ANSWER!

Derive's graphics can really help merge ideas.

4) If $y = 5\sin(x) + 2$, determine the minimum and maximum y-values.

WORKBOOK

$5\sin(x)+2$

$5\sin(x)=y-2$

$\sin(x)=\frac{y-2}{5}$

$-1\le\frac{y-2}{5}\le 1$

$-5\le y-2\le 5$

$-5\le y-2$

$-5+2\le y$

$y\ge -3$

$y-2\le 5$

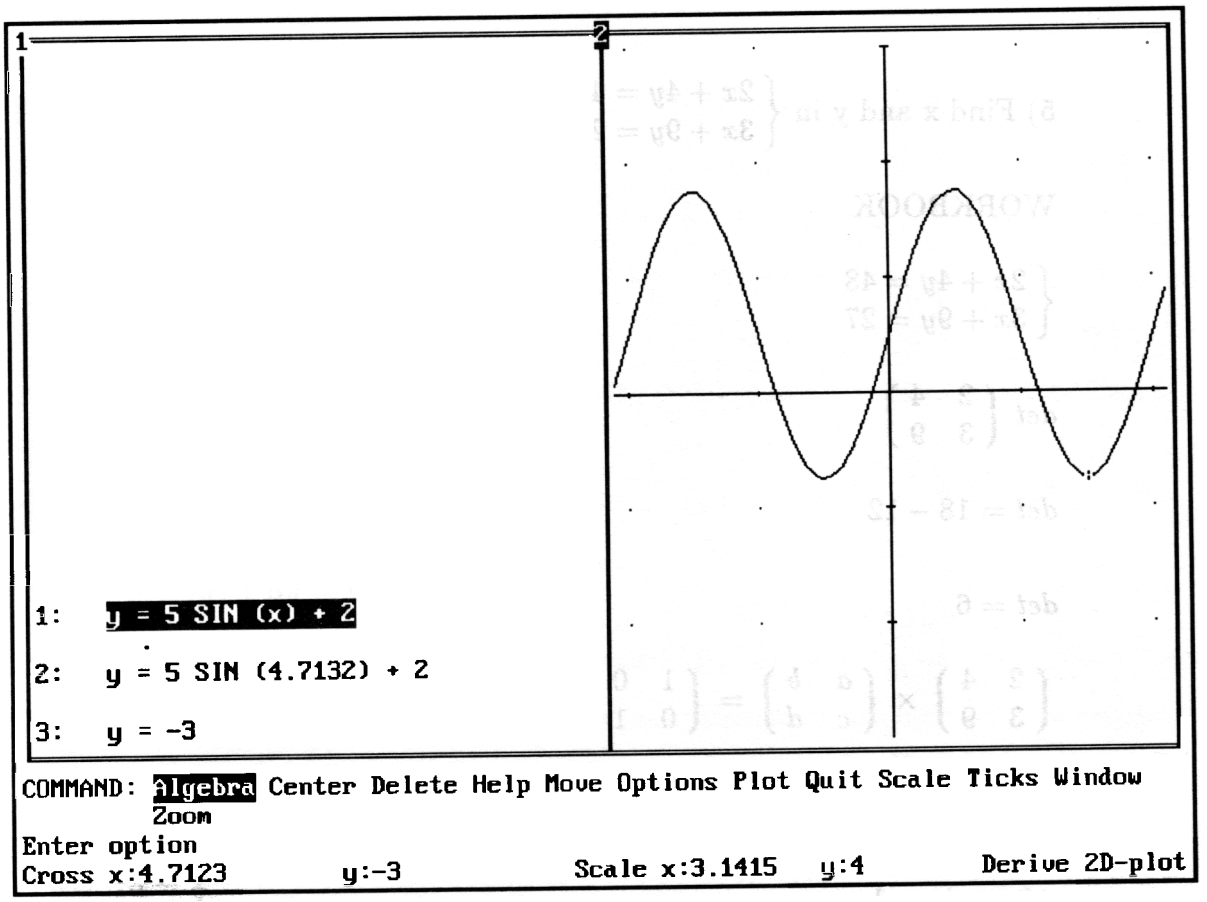
$y\le 5+2$

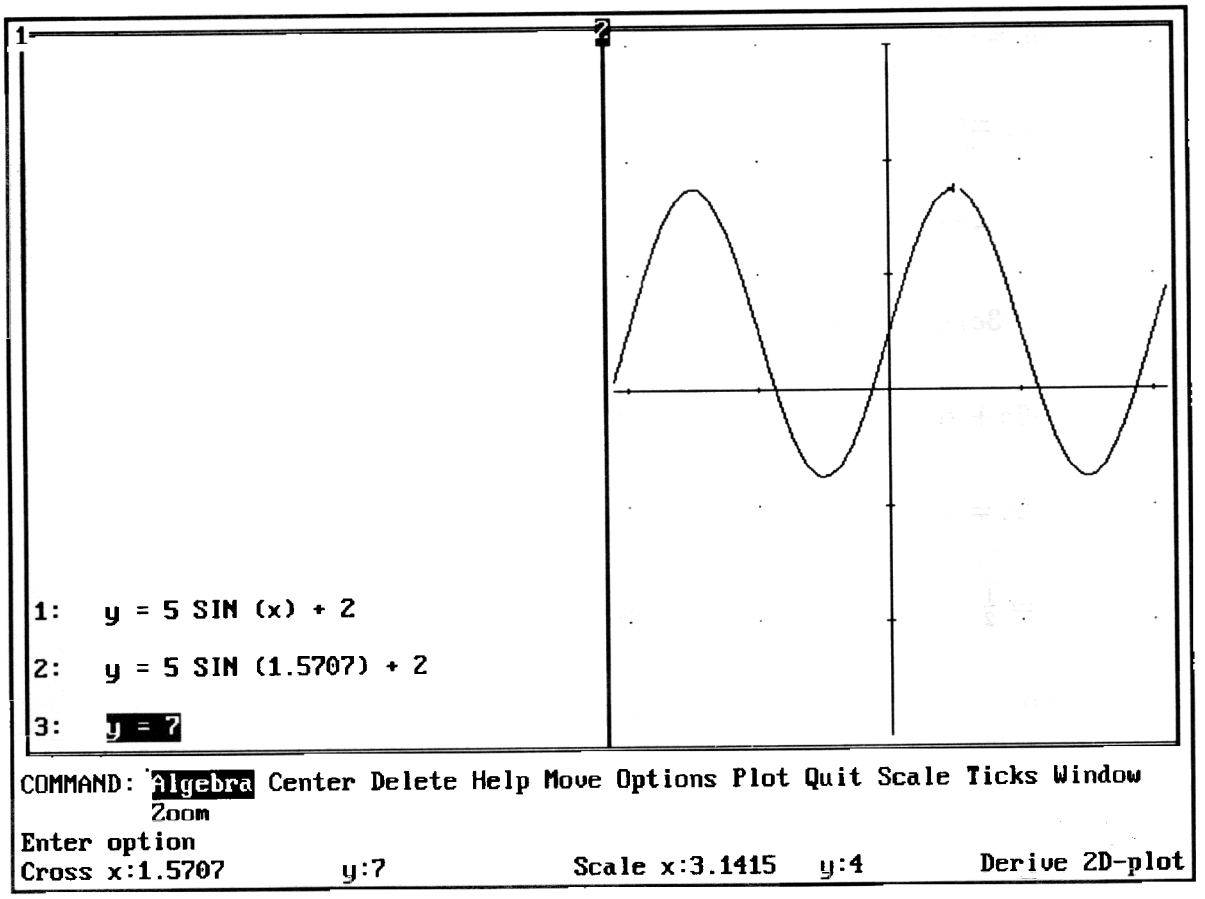
$y\le 7$

$[-3;7]$

DERIVE

$1:\ y=5\sin(x)+2$


1
2
1: y = 5 SIN (x) + 2
2: y = 5 SIN (4.7132) + 2
3: y = -3
COMMAND: Algebra Center Delete Help Move Options Plot Quit Scale Ticks Window
Zoom
Enter option
Cross x:4.7123 y:-3 Scale x:3.1415 y:4 Derive 2D-plot


1
2
1: y = 5 SIN (x) + 2
2: y = 5 SIN (1.5707) + 2
3: y = 7
COMMAND: Algebra Center Delete Help Move Options Plot Quit Scale Ticks Window
Zoom
Enter option
Cross x:1.5707 y:7 Scale x:3.1415 y:4 Derive 2D-plot

5) Find x and y in $\begin{cases} 2x+4y=48 \\ 3x+9y=27 \end{cases}$

WORKBOOK

$$\begin{cases} 2x+4y=48 \\ 3x+9y=27 \end{cases}$$

$$\det\begin{pmatrix} 2 & 4 \\ 3 & 9 \end{pmatrix}$$

$$\det = 18-12$$

$$\det = 6$$

$$\begin{pmatrix} 2 & 4 \\ 3 & 9 \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2a+4c & 2b+4d \\ 3a+9c & 3b+9d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} 2a+4c=1 \\ 3a+9c=0 \end{cases}$$

$$3a=-9c$$

$$a=\frac{-9c}{3}$$

$$a=-3c$$

$$2\cdot(-3c)+4c=1$$

$$\text{-}6c+4c=1$$

$$-2c=1$$

$$c=-\frac{1}{2}$$

So: $a=-3\cdot\left(\frac{-1}{2}\right)$

$$a = \frac{3}{2}$$

$$\begin{cases} 2b + 4d = 0 \\ 3b + 9d = 1 \end{cases}$$

$$2b = -4d$$

$$b = \frac{-4d}{2}$$

$$b = -2d$$

$$3 \cdot (-2d) + 9d = 1$$

$$-6d + 9d = 1$$

$$3d = 1$$

$$d = \frac{1}{3}$$

So, $b = -2 \cdot \left(\frac{1}{3}\right)$

$$b = \frac{-2}{3}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{-2}{3} \\ \frac{-1}{2} & \frac{1}{3} \end{pmatrix} \times \begin{pmatrix} 48 \\ 27 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{2} \times 48 + \frac{-2}{3} \times 27 \\ \frac{-1}{2} \times 48 + \frac{1}{3} \times 27 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 54 \\ -15 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Or you have another way to reach the same answer:

$$\begin{cases} 2x + 4y = 48 \\ 3x + 9y = 27 \end{cases} \Rightarrow 2x = 48 - 4y \Rightarrow x = \frac{48 - 4y}{2}$$

REPLACING:

$$3\times\frac{48-4y}{2}+9y=27$$

$$\frac{144-12y}{2}+9y=27$$

$$72-6y+9y=27$$

$$3y=27-72$$

$$3y=-45$$

$$y=-15$$

$$2x+4y=48$$

$$2x-60=48$$

$$2x=48+60$$

$$x=\frac{108}{2}$$

$$x=54$$

DERIVE

$$1:\begin{pmatrix}2x+4y=48\\3x+9y=27\end{pmatrix}$$

$$2:\ \det\begin{pmatrix}2&4\\3&9\end{pmatrix}$$

$$3:\ 6$$

$$\begin{pmatrix}2&4\\3&9\end{pmatrix}^{-1}\times\begin{pmatrix}48\\27\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}54\\-15\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$$ 5 STEPS !!!

This technique motivated students. Derive is very useful, but workbooks are still important. Teachers can merge these two things. The students became very participative

and concentrated. They lost their math fears. Mechanical calculus is boring but necessary in the begining. But once you understand this process, Derive permits an exploration without mistake trauma. They make conclusions and constantly question themselves. Teachers are there to help, but they have to control their wish to help the students continuously, giving them the ready answers. Instead, give them a clue answering with another question. Remember that the learning process is the most important thing.

4 - Conclusion

Pedagogically speaking, a student's tendency is to be practical and objective. However, the teaching counterpart is to emphasize that facts are moved by concepts. Thus computer algebra appears as a benefit of our age for showing them the face of an idea instead of losing time in dusty calculations. Derive is the cheapest way to put them in contact with mathematical reality and their abstract and logical thoughts.

The Algebraic Computation's pedagogical power allows the return of a medieval tutorial system, i.e., material subject is given to students for improving

themselves, while teachers must serve as guiding educators.

It is just the begining for us, in Brasil, but our future will be really better.

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References

[1] Harper D.; Wooff C.; Hodaknison D. "A Guide to Computer Algebra Systems", Wiley 1991.

[2] User Manual Derive version 2.0.

[3] Piaget J.; Inhelder B., " Introduction à L' Épistémologie Génétique, tomo 1: La Pensíe Mathématique. Paris, P. U. F., 1950.

[4] De La Logique de L' Enfant à la Logique de L' Adolescent, Paris, P. U. F., 1955.

[5] Papert S. "Mindstorms - Children, Computers and Powerful Ideas". Basic Books - Inc 1980.

[6] Machado, A.S., "Matemática: Temas e Metas", Editora Atual, 1988.

[7] Boldrini, J.L. "Álgebra Linear", Editora Harbor, 1980.