BACK

Computer Aided Mathematics in School

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Introduction

I confess I am fascinated doing mathematics with computer support and I try to convince my students of this way to learn maths. Therefore in recent years I have tried to find out where in my lessons it seems reasonable to use computers. Some of these attempts I would like to present and I hope they are of interest to you.

I believe the regular use of computers in maths lessons has considerable methodical and didactical consequences. The most important from my point of view are:
- computers make students more independent of teachers,
- computers allow students to work at their own speed,
- computers enable students to make their own experiences in fields of mathematics,
- computers call for curricular changes, because some subjects become unecessary and others can be added.

These are positive aspects of the use of computers. But they can also be to the disadvantage of students. They must not forget how to do numerical calculation without computers. They must not rely blindly on them and must not give the responsibility for the result and their work to the computers. For that reason computers may be used only to complete regular lessons.

However there are goals in maths lessons which you can reach easier or only with computers. Here is a list of those goals.

1. To practise mathematical methods without the drudgery of performing long mathematical calculations.
+++ * *algorithm for systems of linear equations*
+- * *analysis of functions*

2. To visualize mathematical relations
- * *vector calculus (straight lines, planes, spheres and their intersections in a 3-dimensional space)*
+++ * *graphical display of functions*

3. To gain experiences from many examples
+ * *trim and practice*
++ * *to explore the different solutions of systems of linear equations*

4. To find out mathematical structures or methods by means of own experiments
- * *elementary geometry*
++ * *coordinate transformations*
+ * *to find out the characteristics of classes of functions (rational functions, exponential functions, etc.)*

5. To treat problems which require extensive numerical calculations
++ * *to calculate zeros of functions by iteration*
+ * *fractal geometry (iteration with complex numbers)*

Some of the mentioned subject-matters I found worthwhile to repeat and others not. I marked my personal view by plus and minus signs. Now I would like to inform you in more detail about lessons on "systems of linear equations" and "iterations".

Solving systems of linear equations with the support of the computer

Systems of linear equations are taught in the German curriculum in class 8 (13 year old pupils). It is well known that this is a different matter at that age. Pupils have to learn on one hand the application of the algorithm and on the other hand to carry out a long calculation without making mistakes. By means of suitable software one can separate these two steps. Looking at the solution of a system of 2 linear equations with 2 variables we realize which manipulations must be done by the pupils. Performing the Gauß-Jordan method the solution can be obtained by multiplying and dividing particular equations with integer numbers and by adding two equations. (Substitution will not be introduced because *then* the solving algorithm is the same for two, three or more equations.) A simple program which does this manipulation under the supervision of the pupils will be demonstrated now.

In class 12 vector calculus is treated. At that time the program is used again for introducing linear equations. But in this course the procedure of solution is of no interest anymore. (It should be known.) Now we are interested in the solution and its interpretation from an algebraic and geometric point of view. Therefore the *DERIVE* function ROW-REDUCE fits very well into this concept. It delivers the same result but in one step. Students enter the coefficients of the system or more precisely the extended matrix of the system. *DERIVE* reduces the matrix to the "Row Echelon form", which allows you to specify the solution. (This is not trivial at all for students.)

This tool enables the students to solve many systems of equations. From this experience they can classify the different types of solutions which are possible. Equations with parameters can be treated. This gives a very thorough insight into the theory of linear equations. This will be demonstrated briefly.

In my opinion the treatment of systems of linear equations with computer support is to the advantage of students. In particular the less able students can do the same job as the good ones, because the performing of calculations - their weak point - is done by the computer. In lessons there is more time to discuss the mathematical theory because the numerical problems are reduced.

Approximate calculation of zeros of real functions

The analysis course in class 11 includes as an important item the determination of zeros of functions. If there is no closed solution to an equation, an iteration may be used to find an approximate solution.

In that course I explained the general method of iteration in the classroom. There they have also performed at least one iteration with their electronic calculator. At that stage the convergency of the iteration sequences has not been discussed, because I want them to find out these conditions by their own explorations in the computer room.

With the *DERIVE* functions ITERATES, VECTOR, and ELEMENT it is easy to provide students with tools to investigate iterations with different functions and different initial values. Students can use three tools:
(1) Display of the elements of the sequence $\{x_0,x_1,x_2,...\}$.
(2) Plotting the sequence $\{x_0,x_1,x_2,...\}$ on the x axis from which the students can see whether or where the sequence accumulates.
(3) Plotting the elements of the sequence $<x_i>$ as a function of the order i of iteration.

As a first example I used the function defined by $f(x):=x^2-x-1$. Why a quadratic equation? Which solution is easy to obtain? Three reasons: In the first application it is useful to know the solution. There are numerous equations of the form x=phi(x). This equation f(x)=0 plays a particular role in fractal geometry.

When students are familiar with the tools they shall find out:
- Which of the two zeros are approximated and why?
- Why some functions generate convergent sequences and others not?
- How the convergency depends on the initial value?

(If they do not find the answers, I will introduce the function $phi(x):=x+k(x^2-x-1)$ and ask them to vary k. Then they definitely will find the slope of the curves to be responsible for convergency.) The lesson ends with the proof of the conditions of convergency.

Iteration with complex numbers

The tools shown also can be used to introduce fractal geometry, dynamic systems or chaos theory.

The students did not find any convergent sequences with the computer when using the function phi(x):=x^2-1. The sequences are either not limited or they have the two attractors -1 and 0. The reason is well known because the absolute value of the first derivative is too large. But there is an infinity of convergent sequences for initial values which are irrational numbers. These numbers cannot be realized on a computer but only rational approximations of these numbers. (This is the first contact with chaos. An arbitrary small deviation from the initial value gives a totally different solution.)

Now the students should extend their investigation to the plane and perform iterations with complex numbers. The tool enables them to display the elements of the sequence in the complex plane.

They have the chance to find out which numbers belong to the Julia set and the Mandelbrot set. With *DERIVE* functions they can only get a rough estimate of the extension of these sets. We finished the lesson with an application of PD software which allows the display of Julia sets in detail.