BACK

THE USAGE OF DERIVE IN MATHEMATICS BY FIFTEEN-YEAR-OLD PUPILS.

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1 Description of the situation

My class consists of 14 pupils, 2 girls and 12 boys. In their third form they chose informatics as a compulsary subject and therefore have some knowledge of typewriting. They are used to working with computers as they all have a computer of their own. Presently the boys pass a phase of puberty and mostly do not show any commitment. During the first few months, they could not cope with the fact of being in a group of only 14 pupils. "Being not able to hide" was quite some handicap to them and they felt like being "outsiders" among the other pupils of their age.

2) Using Derive

At the end of September I began to introduce Derive in my maths lessons. We started with simplifying and solving equations. A disc containing the software was handed out to the pupils to allow them to work at home as well. I composed a set of lecture notes describing the most important instruc-tions and explaining them in examples. Later on they were given copies of the reference book.

2.a) Derive in lessons

As the room the computers are placed in is not used during my maths lessons, I may use it even if needing it just for a few minutes. Lessons held in the computerroom require even more discipline from the pupil than usually. Behind the monitors they can hide and are likely to try out other kinds of soft-ware or to experience "more interesting" possibilities of Derive.
The monitor cuts off the teacher's contact to the pupils. Sometimes the teacher has to stay in front of the classroom, where the overhead projector is placed, instead of staying at the back of the classroom to keep up control. While the teacher is helping one of the pupils the others instantly start doing anything else. Even in taking notes there is a lack of disci-pline: The pupils sometimes are not able to decide whether to write something up, to type it or to print it out. They also do not always keep their notes in the right order.
The pupils get to know about arithmetic methods, but do not have to go through endless exercises as the computer takes this part (White Box - Black Box - Method).
Consequently there remains more time for interpretation.
Here is a simple example of how the pupils work on Derive:

$$2x-3y=5$$
$$3x+2y=1$$

1) Gauss' Method of Elimination:
(2x-3y=5)*2+(3x+2y=1)*3 and SIMPLIFY supplies the solution.

2) Substitution:
SOLVE expresses one variable by the other in one of the equations and MANAGE SUBSTITUTE replaces it in the other equation. SOLVE solves the remaining equation.

3) The input of [2x-3y=5, 3x+2y=1 instantly supplies the solution.

In home-exercises and tests the way of solution must be correctly indicated. By this I check whether the pupils have understood the different methods. Method number 3) shortens the solutions of complex tasks.
Even in mathematics one may be creative:

Examining the function $f(x,y) = x.\left(1-\frac{1}{y}\right.$ requires quite some creativity

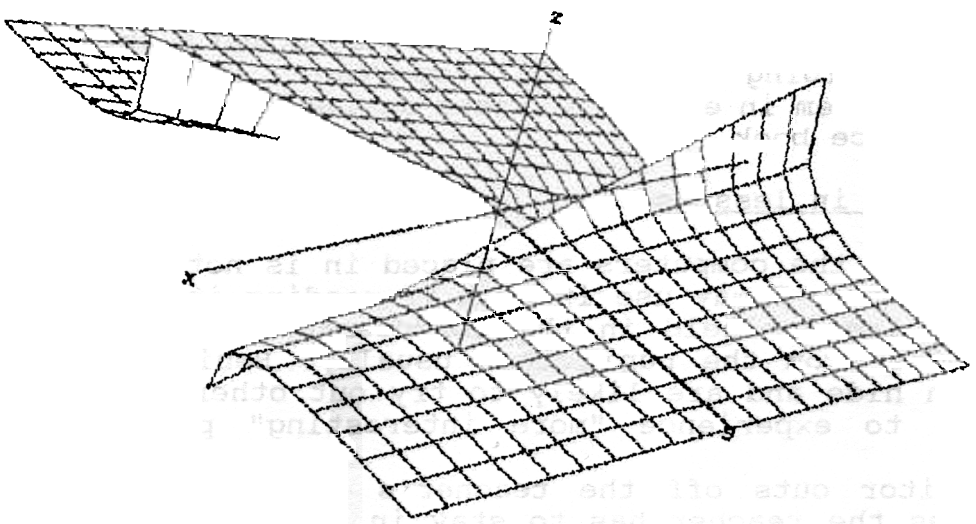

Abb. 1 Graphic of the function f(x,y)

Graphic depiction provides massive help for the pupils. A group of average ability found out the following:
Firstly they were impressed by the 3-dimensional depiction. They had some problems with reading information from the distorted graph, though. They started to keep x constant and to delimit the set of numbers. They found out that the graph was split up in two parts as there were two variables. They set x=3 and saw that the function described a hyperbola the branches of which were constantly increasing. If x<0, the monotony of the branches changes. They found out that lines of intersection which were parallel to the yz-level were branches of an

hyperbola, that x=0 or y=1 results in f(x,y)=0. In the following they kept y constant which made it more difficult to distinguish the two cases. With a little help from me they examined the graph with y=2 and y=0.5. They saw that the lines of intersection parallel to the xz-level were straight lines. They also tried to find out for which pairs of numbers f was above or below zero. After this detailed examination of the graph, they were more able to imagine 3-dimensional pictures. The ability of imagination is improved by sort of these examples.
There may also be problems in other subjects like, e.g., physics, in which arithmetical operations are performed which the pupils are not yet able to do and Derive is not used.

2.b) Derive at home-exercises:

It must be clearly expressed whether the pupils are to use Derive or not. The printing out needs a lot of time which seems to be a problem.

2.c) Derive at written exams:

There exists a special disc for exams that is labelled with the pupils name. There is also a seating plan for each examination. The first exams were quite nerveracking, for the pupils as well as for me. The pupils feared that the computer might not function well or that they might make mistakes when saving their writings on the disc. By now we have become accustomed to this situation. Only once there were problems with one of the computers. The text the teacher hands out to the pupils is far longer than usually, which means that the pupils need more time for reading it. On the other hand the operations are performed faster and the pupils do not have to check their calculations any more. As the pupils and their parents were not able to overestimated the workingtime, they felt to be at a disadvantage compared to the pupils of the other classes. Initially the pupils preferred their pocket calculator when performing secondary calculations, but by now they are hardly used any more. It took some time as well until the pupil used the operation SOLVE when allowed to. From force of habit they solved the equations themselves by which they lost a lot of time. After the examination I printed out what they had saved to their discs, which was quite a lot of work but made working more comfortable. I have performed only three written exams with the use of Derive, so far. If an arithmetical method is still "White Box", the example must be solved without Derive. There have hardly been any arithmetical errors. Interpreting, proving and solving of problems is even more difficult to the pupils though. Firstly they have problems in reading and understanding the text and then they even have to explain it. Many mistakes result from the inexact way of expression. They want to say the right thing but are not able to express it correctly.

Which were the mistakes they made?

1) Perform the calculation with the help of Derive and save the result as BSP3A!
Write the set of possible results to your writing book!

$$\left|3-\frac{x}{2}\right| > 5$$

The pupils first had to reduce the inequation to:

$$3-\frac{x}{2}>5 \vee 3-\frac{x}{2}<-5$$

Then they were allowed to solve the inequations with SOLVE. Mistakes were made at the reduction of the inequation.

2) Perform the calculation with the help of Derive and save the result as BSP3B! Use substitution to solve the system of equations!

$$x - y = 2b$$
$$bx + ay = a^2+b^2$$

Three of the pupils did not read the text properly and used Gauss' Method of Elimination instead of substitution. These mistakes have nothing to do with Derive itself. They occur at "normal" examinations, as well. At the last examination the solving of equations with Derive was not the least problem any more. They knew by then how to use it properly.

3. SCHULARBEIT, am 23.1.92. 5. KLASSE

1) a) Check, if the following are functions. Explain in case they are not.

f: $A \begin{pmatrix}1\\2\\3\end{pmatrix} \begin{pmatrix}2\\5\\7\end{pmatrix} B$ g: $A \begin{pmatrix}1\\2\\3\end{pmatrix} \begin{pmatrix}2\\5\end{pmatrix} B$ h: $A \begin{pmatrix}1\\2\\3\end{pmatrix} (4 \quad B$

b) Restrict domain R in such a way that the equation $2y=xy+2x$ turns into a function like: $f: x \mapsto y$.

c) Prove that the function $f(x) = -5x-2$ is strictly monotonically decreasing.

2) s_1 and s_2 are displacement/time functions.
s_1: $s = 4t$
s_2: $s = 18 - 5t$

a) Write a text for the assumption!
b) Solve the equation system, above using Derive and save under BSP2B! Write the solution of your problem in your notebook!

c) What will be changed in your text, if s_2 is to be changed in the following way:
s_2: $s = 18 - 5(t+0.6)$

3) a) $f(-3)=2$ $\quad A = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Find a term of the linear function!

b) $g(x) = -3x+2 \qquad x = -2 \pm 10\ \%$.
Find an interval for the values of the function.
By how many precents do the values differ from f(-2)?
Use Derive and save under BSP2B! Write the solution in your notebook!

c) Explain why $k = \dfrac{f(x_2)-f(x_1)}{x_2-x_1}$! Make a sketch!

4) a) g: $X = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + s.\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ $\quad$ h: $X = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + t.\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$

Check if g intersects with h.

b) Find parameters of the straight line

a: passing $P = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ parallel to the 1st axis

b: passing $R = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ parallel to the 2nd axis

c: passing the origin parallel to h: $X = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + s.\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

c) The vectors a and b depend linearly. Find the missing coordinates.

$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ a_3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b_1 \\ 2 \\ 4 \end{pmatrix}$

d) Check if A, B, C are on a straight line!

$A = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 6 \\ -1 \end{pmatrix}$

e) Find the intersection of the diagonal of the quadrilateral ABCD:

$A = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 9 \end{pmatrix} \quad D = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

In this test the solving of inequations and of equation systems were already part of the "Black Box". Therefore Derive could be used for problems, which reduces the time necessary for solving them.
Using vectors was "White Box"; that is why Derive was not used.
In my opinion the examinations have become more diffi-cult. The pupils do not have to show their ability of calcu-lating, but have to understand mathematics. This could as well be achieved without Derive, though.
For using Derive at exams it is necessary that each pupil has a computer and software of his own. This has not been possi-ble in larger groups so far. Pupils of an average class would have to come to school with a notebookcomputer, and would not have to change the classroom. They could use Derive in all other subjects, as well.

3) <u>What will be changed in mathematics by using Derive?</u>

a) More time to interpret mathematical problems.
b) New mathematical topics.
c) More projects, also envolving other subjects.
d) History of mathematics.

Our scholar system comprehends three main types of subjects:
* Subjects providing general education
* Languages
* Mathematics

Mathematics is quite isolated and at best connected with natural sciences. It's mostly considered to be no more than an unpleasant duty.

Which are the reasons for this situation?

The pupil is taught arithmetical methods and believes that they have always existed. At the utmost some pupils know the names of some Greek mathematicians, but no more. There has not been any time, yet, to show that mathematics is constantly developing and that mathematics is an enormous part of our culture and has a huge influence on its evolution.Mathematics should not be seen as isolated, but as an important part of our general education.